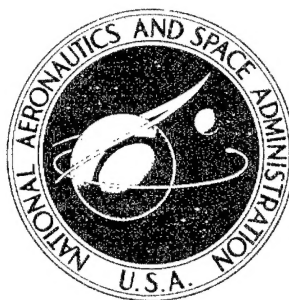


**NASA CONTRACTOR  
REPORT**



064259

NASA CR-458

NASA CR-458

**DISTRIBUTION STATEMENT A**  
Approved for Public Release  
Distribution Unlimited

AD-7320

**COMPUTATIONAL METHODS FOR  
CALCULATION OF RATES OF  
EVAPORATION IN THE Ta-Hf-C SYSTEM**

*by Larry Kaufman and Gerald Stepakoff*

Prepared under Contract No. NAS 3-4910<sup>13</sup> by

MANLABS, INC.

Cambridge, Mass.

for Lewis Research Center

20020411 095

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • APRIL 1966<sup>10</sup>

COMPUTATIONAL METHODS FOR CALCULATION OF RATES  
OF EVAPORATION IN THE Ta-Hf-C SYSTEM

By Larry Kaufman and Gerald Stepakoff

Distribution of this report is provided in the interest of information exchange. Responsibility for the contents resides in the author or organization that prepared it.

Prepared under Contract No. NAS 3-4910 by  
MANLABS, INC.  
Cambridge, Mass.

for Lewis Research Center

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

# LIST OF FIGURES

| <u>Fig. No.</u> |  | <u>Page</u> |
|-----------------|--|-------------|
| 1               | Vaporization of (Ta-Hf)C at 2200°K and 50% carbon .  | 13          |
| 2               | Vaporization of (Ta-Hf)C at 2200°K and 49% carbon .  | 14          |
| 3               | Vaporization of (Ta-Hf)C at 2200°K and 46% carbon .  | 15          |
| 4               | Vaporization of (Ta-Hf)C at 2200°K and 41% carbon .  | 16          |
| 5               | Vaporization of (Ta-Hf)C at 2600°K and 50% carbon .  | 17          |
| 6               | Vaporization of (Ta-Hf)C at 2600°K and 49% carbon .  | 18          |
| 7               | Vaporization of (Ta-Hf)C at 2600°K and 46% carbon .  | 19          |
| 8               | Vaporization of (Ta-Hf)C at 2600°K and 41% carbon .  | 20          |
| 9               | Vaporization of (Ta-Hf)C at 2800°K and 50% carbon .  | 21          |
| 10              | Vaporization of (Ta-Hf)C at 2800°K and 49% carbon .  | 22          |
| 11              | Vaporization of (Ta-Hf)C at 2800°K and 46% carbon .  | 23          |
| 12              | Vaporization of (Ta-Hf)C at 2800°K and 41% carbon .  | 24          |
| 13              | Vaporization of (Ta-Hf)C at 3000°K and 50% carbon .  | 25          |
| 14              | Vaporization of (Ta-Hf)C at 3000°K and 49% carbon .  | 26          |
| 15              | Vaporization of (Ta-Hf)C at 3000°K and 46% carbon .  | 27          |
| 16              | Vaporization of (Ta-Hf)C at 3000°K and 41% carbon .  | 28          |
| 17              | Computed initial rates of vaporization at 2200°K .   | 31          |
| 18              | Comparison of observed (average) rates of<br>vaporization with computed (initial) rates of<br>vaporization at 2600°K . . . . .   | 32          |
| 19              | Computed initial rates of vaporization at 2800°K . .   | 33          |
| 20              | Computed initial rates of vaporization at 3000°K . .   | 34          |
| 21              | Calculated compositional dependence of the vapor<br>pressures of Ta and C over the TaC <sup>σ</sup> phase at<br>2600°K . . . . . | 36          |

# LIST OF TABLES

|   |   |    |
|---|---|----|
| 1 | Summary of Numerical Values Required for<br>Vapor Pressure Computations . . . . . | 11 |
|---|---|----|

## TABLE OF CONTENTS

|  | <u>Page</u> |
|--|-------------|
| I. INTRODUCTION . . . . .  | 1           |
| II. EXTENSION OF THE SCHOTTKY-WAGNER<br>MODEL TO TERNARY COMPOUNDS . . . . . | 3           |
| III. APPLICATION OF MODEL TO Ta - Hf - C SYSTEM                              | 10          |
| IV. CONCLUSIONS . . . . .  | 37          |

## I. INTRODUCTION

*Stu* In recent studies of phase equilibria in refractory compound systems at ManLabs, explicit methods have been developed for predicting the vapor pressures of the individual component elements over NaCl type metal carbides. <sup>(1, 2)</sup> These methods have been developed and applied to a series of refractory binary compounds which are stable over a wide range of temperature and composition. Comparison of the predicted vapor pressures with experimental measurements of congruent vaporization in HfC and ZrC yields excellent results. In addition, comparison of predicted congruently vaporizing composition in the niobium-carbon system and the observed congruency agree quite well.

On the basis of this experience, it seems worth while to extend the method, derived for binary systems, to a ternary metal-metal-carbide compound (Ta-Hf)C, which has the same stoichiometry and crystal structure as the binary carbides discussed earlier.

Under these circumstances, a theoretical study of (Ta-Hf)C was undertaken. The objectives of the present study were to

- a) Apply the Schottky-Wagner model, used previously to describe binary compounds, <sup>was applied</sup> to the ternary (Ta-Hf)C in order to generate equations capable of predicting the vapor pressures of tantalum, hafnium, and carbon over (Ta-Hf)C as a function of temperature and composition. *Fig. 2*

- Some related*
- b) Relate the computed vapor pressure curves to rates of vaporization characteristic of specific compositions through the Langmiur equation.
  - c) Present explicit equations for the vapor pressure and rate of vaporization of each component as a function of temperature and composition.

and

- d) Carry out explicit numerical evaluation of the vapor pressure and rate of evaporation of each component at 2200°K, 2600°K, 2800°K and 3000°K and at carbon levels of 41%, 46%, 49% and 50% respectively for comparison with experimentally observed results.

Subsequent portions of this report detail the methods used to accomplish the objectives described above.

## II. EXTENSION OF THE SCHOTTKY-WAGNER MODEL TO TERNARY COMPOUNDS

In order to extend the treatment of Kaufman and Clougherty,<sup>(1)</sup> performed for binary compounds, to the ternary case, we consider a system A-B-C containing a ternary compound having the  $\sigma$  crystal structure. The composition is specified by setting  $(1-x-y)$  equal to the atom fraction of A,  $x$  equal to the atom fraction of B, and  $y$  equal to the atom fraction of C. We consider a case where two sublattices exist, the A and B atoms occupying one sublattice, while the C atoms occupy the other (i.e. A = Ta, B = Hf, C = carbon). If stoichiometry corresponds to  $y = y_0$  and the total number of sites, filled and unfilled, is  $N_s$  then,

$$N_{sA} = \text{Number of A sites} = (1-x-y) (1-y_0) (1-y)^{-1} N_s$$

$$N_{sB} = \text{Number of B sites} = x(1-y_0) (1-y)^{-1} N_s$$

$$N_{sC} = \text{Number of C sites} = y_0 N_s$$

while

$$N_A = \text{Number of A atoms} = (1-x-y)N$$

$$N_B = \text{Number of B atoms} = xN$$

$$N_C = \text{Number of C atoms} = yN$$

where  $N$  is Avogadro's number.

If the ratio of A atoms on A sites,  $N_{A1}$ , to A atoms on B sites,  $N_{A0}$ , is equal to the ratio of A sites to B sites (ditto for the B atoms) then

$$N_{A0} = \text{Number of A atoms on B sites} = x(1-x-y)N(1-y)^{-1}$$

$$N_{A1} = \text{Number of A atoms on A sites} = (1-x-y)^2 N(1-y)^{-1}$$

and

$$N_{B0} = \text{Number of B atoms on A sites} = x(1-x-y)N(1-y)^{-1}$$

$$N_{B1} = \text{Number of B atoms on B sites} = x^2N(1-y)^{-1}$$

In line with the standard state convention adopted earlier, <sup>(1)</sup> the free-energy per gram atom  $F^{\sigma}$  is given by equation (1)

$$F^{\sigma} = (1-x-y)F_A^{\circ} + xF_B^{\circ} + yF_C^{\circ} + \left(\frac{N_s}{N}\right) \Delta F^{\sigma} + \frac{N_{A+}}{N} F_{A+} + \frac{N_{B+}}{N} F_{B+} + \frac{N_{C+}}{N} F_{C+} \\ + \frac{N_{A0}}{N} F_{A0} + \frac{N_{B0}}{N} F_{B0} - kT \ln W_p \quad (1)$$

In Eq. 1,  $F_A^{\circ}$ ,  $F_B^{\circ}$ , and  $F_C^{\circ}$  are the free energies of pure A, B, and C at the temperature in question, where

$$F_A^{\circ} [0^{\circ}\text{K}] = F_B^{\circ} [0^{\circ}\text{K}] = F_C^{\circ} [0^{\circ}\text{K}] = 0 \text{ at one atmosphere} \quad (2)$$

is the reference state. Moreover,  $\Delta F^{\sigma}$  is the free energy of formation of the ternary compound for a given value of  $x$  and  $y_0$ . The free energies of A atoms on B sites and B atoms on A sites are given by  $F_{A0}$  and  $F_{B0}$ , while  $F_{A+}$ ,  $F_{B+}$ , and  $F_{C+}$  are the free energies of formation for A, B, and C vacancies. The numbers of A, B, and C vacancies, which appear in Eq. 1 are given as follows:

$$N_{A+} = \text{Number of vacant A sites} = (1-x-y)(1-y_0)N_s(1-y)^{-1} = (1-x-y)N$$

$$N_{B+} = \text{Number of vacant B sites} = x(1-y_0)N_s(1-y)^{-1} - xN$$

$$N_{C+} = \text{Number of vacant C sites} = y_0 N_s - yN$$

The final term to be evaluated in Eq. 1 is the thermodynamic probability factor  $W_p$  which is given by,

$$W_p = \frac{N_s C!}{N_{C+}! N_C!} \cdot \frac{N_s A!}{N_{A+}! N_{A1}! N_{B0}!} \cdot \frac{N_s B!}{N_{B+}! N_{B1}! N_{A0}!} \quad (3)$$



Making the appropriate substitutions for the  $N_g$  and applying Stirling's formula and substitution into Eq. (1) yields for the case  $y_0 = 1/2$

$$\begin{aligned}
 F^{\sigma} = & (1-x-y)F_A^{\circ} + xF_B^{\circ} + yF_C^{\circ} + z(\Delta F_{AC}^{\sigma} + x(1-y)^{-1}(\Delta F_{BC}^{\sigma} - \Delta F_{AC}^{\sigma})) \\
 & + (0.5z + y - 1)(1-y)^{-1}((1-x-y)F_{A+} + xF_{B+}) + (0.5z - y)F_{C+} + x(1-x-y)(1-y)^{-1}W \\
 & + RT(-z \ln 0.5z + y \ln y + x \ln x + (1-x-y) \ln (1-x-y) \\
 & + (0.5z - y) \ln (0.5z - y) + (0.5z + y - 1) \ln (0.5z + y - 1)) \quad (4)
 \end{aligned}$$

where  $z = \text{ratio of sites to atoms} = N_g/N$ ,  $W = F_{A0} + F_{B0}$ , and the free energy of formation of the ternary compound  $\Delta F^{\sigma}[x, y_0]$  has been approximated by a linear combination of the free energies of formation of stoichiometric AC and BC which represent TaC and HfC. Under these conditions Eq. (4) reduces to

$$\begin{aligned}
 F^{\sigma} = & (1-x)F_A^{\circ} + xF_B^{\circ} + 2(\Delta F_{AC}^{\sigma} + x(\Delta F_{BC}^{\sigma} - \Delta F_{AC}^{\sigma}) + \frac{1}{2}F_{C+}) + Wx(1-x) \\
 & + RT(x \ln x + (1-x) \ln (1-x)) \quad (5)
 \end{aligned}$$

for the case where  $y = 0$  and  $N_g/N = 2$  which corresponds to a substitutional solid solution of A in B with the C sublattice empty. Eq. (5) is recognizable as being the regular solution approximation for substitutional solid solutions. In the other limits where  $x = 0$  or  $1-x-y = 0$  (i.e. no B or A atoms are present), Eq. (5) reduces to the equation for the binary compound identical with the expression derived earlier.<sup>(1)</sup>

The next step in the derivation is to fix the composition and minimize the free energy at constant temperature and pressure by letting the volume (i.e. the total number of sites) vary. This procedure is performed by setting the derivative of  $F^{\sigma}$  (Eq. 4) with respect to  $z$  at constant  $x, y$ , and  $T$  equal to 0.

The result is

$$2(\Delta F_{AC}^{\sigma} + x(1-y)^{-1} (\Delta F_{BC}^{\sigma} - \Delta F_{AC}^{\sigma})) + (F_{A+} + x(1-y)^{-1} (F_{B+} - F_{A+})) + F_{C+} \\ = -2RT \ln 2\alpha \quad (6)$$

where

$$-2RT \ln 2\alpha = RT \ln (0.5z)^2 (0.5z-y)^{-1} (0.5z+y-1)^{-1} \quad (7)$$

when  $y = y_0 = 1/2$ ,  $\alpha = (z-1)/2z = 0.5(N_s - N)/N_s$ . Thus, as in the binary case,

$\alpha$  is the fractional number of vacant sites at stoichiometry. Substitution of Eqs. 6 and 7 into Eq. (5), i.e. minimization of the free energy with respect to volume, at constant composition, temperature, and pressure, yields

$$F^{\sigma} = (1-x-y)(F_A^{\sigma} - F_{A+}) + x(F_B^{\sigma} - F_{B+}) + y(F_C^{\sigma} - F_{C+}) + x(1-x-y)(1-y)^{-1}W \\ + RT(x \ln x + y \ln y + (1-x-y) \ln (1-x-y) - y \ln (0.5z-y) \\ - (1-y) \ln (0.5z+y-1)) \quad (8)$$

Eqs. 6, 7 and 8 completely define the temperature and compositional dependence of the free energy in terms of the parameters  $F_{A+}$ ,  $F_{B+}$ ,  $F_{C+}$ ,  $\alpha$  and  $W$ .

For the case of carbide type  $\sigma$  phase (NaCl type where the phase boundary on the carbon rich side approaches 50% carbon, i.e.  $y_0 = 1/2$ ), we can approximate<sup>(1)</sup>

$$F_{C+} = -RT \ln 2\alpha = -2\Delta H_{AC}^{\sigma} [0^{\circ}K] - RT \ln 2 \quad (9)$$

where

$$2\Delta H_{AC}^{\sigma} [0^{\circ}K] = 2\Delta H_{AC}^{\sigma} [0^{\circ}K] + x(1-y)^{-1} (2\Delta H_{BC}^{\sigma} - 2\Delta H_{AC}^{\sigma}) \quad (10)$$

$$F_{A+} = -2\Delta H_{AC}^{\sigma} [0^{\circ}K] - RT \ln 2 - 2\Delta F_{AC}^{\sigma} [T] \quad (11)$$

and

$$F_{B+} = -2\Delta H_{BC}^{\sigma} [0^{\circ}K] - RT \ln 2 - 2\Delta F_{BC}^{\sigma} [T] \quad (12)$$

Thus in cases where the free energies of formation of the compounds AC and BC are known, at stoichiometry, as a function of temperature, all the parameters required are specified with the exception of  $W$ .

The final step in the present analysis is the derivation of the partial molar free energies. For the case of a binary system (i.e. if  $y$  were equal to zero) these relations are well known.

$$\overline{F}_A = F - x \frac{\partial F}{\partial x} \quad (13)$$

and

$$\overline{F}_B = F + (1-x) \frac{\partial F}{\partial x} \quad (14)$$

where  $\overline{F}_A$  and  $\overline{F}_B$  are the partial molar free energies of A and B and  $x$  is the atom fraction of B and  $(1-x)$  is the atom fraction of A. The analogous expressions for the ternary case are

$$\overline{F}_A = F - x \left( \frac{\partial F}{\partial x} \right)_y - y \left( \frac{\partial F}{\partial y} \right)_x \quad (15)$$

$$\overline{F}_B = F + (1-x) \left( \frac{\partial F}{\partial x} \right)_y - y \left( \frac{\partial F}{\partial y} \right)_x \quad (16)$$

$$\overline{F}_C = F - x \left( \frac{\partial F}{\partial x} \right)_y + (1-y) \left( \frac{\partial F}{\partial y} \right)_x \quad (17)$$

Since Eqs., 6, 7, and 8 yield

$$\left( \frac{\partial F}{\partial x} \right)_y = -(F_A^0 - F_{A+}) + (F_B^0 - F_{B+}) + W(1-2x-y)(1-y)^{-1} + RT \ln x(1-x-y)^{-1} + 0.5z(1-y)^{-1} (F_{B+} - F_{A+} + 2(\Delta F_{BC}^0 - \Delta F_{AC}^0)) \quad (18)$$

and

$$\left( \frac{\partial F}{\partial y} \right)_x = -(F_A^0 - F_{A+}) + (F_C^0 - F_{C+}) - Wx^2(1-y)^{-2} + 0.5zx(1-y)^{-2} (F_{B+} - F_{A+} + 2(\Delta F_{BC}^0 - \Delta F_{AC}^0)) + RT \ln y(0.5z + y - 1)(1-x-y)^{-1} (0.5z - y)^{-1} \quad (19)$$

Substitution into Eqs., 15, 16, and 17 yields the expression for the partial molar free energies as follows:

$$\begin{aligned}\overline{F}_A^\sigma - F_A^\circ = & -F_{A+} + W x^2 (1-y)^{-2} - 0.5xz (1-y)^{-2} (F_{B+} - F_{A+} + 2(\Delta F_{BC}^\sigma - \Delta F_{AC}^\sigma)) \\ & + RT \ln (1-x-y) (0.5z + y-1)^{-1}\end{aligned}\quad (20)$$

$$\begin{aligned}\overline{F}_B^\sigma - F_B^\circ = & -F_{B+} + W(1-x-y)^2 (1-y)^{-2} + 0.5(1-x-y)z (1-y)^{-2} (F_{B+} - F_{A+} \\ & + 2(\Delta F_{BC}^\sigma - \Delta F_{AC}^\sigma)) + RT \ln x (0.5z + y-1)^{-1}\end{aligned}\quad (21)$$

$$\overline{F}_C^\sigma - F_C^\circ = -F_{C+} + RT \ln y (0.5z-y)^{-1}\quad (22)$$

Since

$$\overline{F}_A^\sigma - F_A^\circ = RT \ln p_A^\sigma [x, y] / p_A^\circ\quad (23)$$

$$\overline{F}_B^\sigma - F_B^\circ = RT \ln p_B^\sigma [x, y] / p_B^\circ\quad (24)$$

and

$$\overline{F}_C^\sigma - F_C^\circ = RT \ln p_C^\sigma [x, y] / p_C^\circ\quad (25)$$

where  $p_A^\sigma [x, y]$ ,  $p_B^\sigma [x, y]$ , and  $p_C^\sigma [x, y]$  are the pressures of A, B, and C respectively over the alloys and  $p_A^\circ$ ,  $p_B^\circ$ , and  $p_C^\circ$  are the corresponding vapor pressures of the pure elements, Eqs. 20 through 25 can be used to compute the vapor pressures of the elements over the alloys.

In particular when  $y = y_0 = 0.5$

$$\begin{aligned}\overline{F}_A^\sigma - F_A^\circ = & -F_{A+} + 4Wx^2 - 2x(F_{B+} - F_{A+} + 2(\Delta F_{BC}^\sigma - \Delta F_{AC}^\sigma)) \\ & + RT \ln (0.5-x) (1-2\alpha) \alpha^{-1}\end{aligned}\quad (26)$$

$$\begin{aligned}\overline{F}_B^\sigma - F_B^\circ = & -F_{B+} + 4W(0.5-x)^2 + 2(0.5-x)(F_{B+} - F_{A+} + 2(\Delta F_{BC}^\sigma - \Delta F_{AC}^\sigma)) \\ & + RT \ln x (1-2\alpha) \alpha^{-1}\end{aligned}\quad (27)$$

and

$$\overline{F}_C^\sigma - F_C^\circ = -F_{C+} + RT \ln 0.5 (1-2\alpha) \alpha^{-1}\quad (28)$$

where  $z$  can be approximated by unity.

When  $y$  is less than 0.5,  $z \approx 2(1-y)$  and  $(0.5z + y - 1)$  is approximately equal to  $4\alpha^2(1-y)^2(1-2y)^{-1}$ . Under these conditions Eqs. 19 through 21 reduce to

$$\begin{aligned} \overline{F}_A^\sigma - F_A^o &= -F_{A+} + Wx^2(1-y)^{-2} - x(1-y)^{-1}(F_{B+} - F_{A+} + 2(\Delta F_{BC}^\sigma - \Delta F_{AC}^\sigma)) \\ &\quad + RT \ln(1-x-y)(1-2y) 4^{-1} \alpha^{-2}(1-y)^{-2} \end{aligned} \quad (29)$$

$$\begin{aligned} \overline{F}_B^\sigma - F_B^o &= -F_{B+} + W(1-x-y)^2(1-y)^{-2} + (1-x-y)(1-y)^{-1}(F_{B+} - F_{A+} + 2(\Delta F_{BC}^\sigma - \Delta F_{AC}^\sigma)) \\ &\quad + RT \ln x(1-2y) 4^{-1} \alpha^{-2}(1-y)^{-2} \end{aligned} \quad (30)$$

and

$$\overline{F}_C^\sigma - F_C^o = -F_{C+} + RT \ln y(1-2y)^{-1} \quad (31)$$

When  $y$  is greater than 0.5,  $z \approx 2y$ , and  $(0.5z - y)$  is approximately  $4\alpha^2 y^2 (2y-1)^{-1}$

and

$$\begin{aligned} \overline{F}_A^\sigma - F_A^o &= -F_{A+} + Wx^2(1-y)^{-2} - xy(1-y)^{-2}(F_{B+} - F_{A+} + 2(\Delta F_{BC}^\sigma - \Delta F_{AC}^\sigma)) \\ &\quad + RT \ln(1-x-y)(2y-1)^{-1} \end{aligned} \quad (32)$$

$$\begin{aligned} \overline{F}_B^\sigma - F_B^o &= -F_{B+} + W(1-x-y)^2(1-y)^{-2} + (1-x-y)y(1-y)^{-2}(F_{B+} - F_{A+} + 2(\Delta F_{BC}^\sigma - \Delta F_{AC}^\sigma)) \\ &\quad + RT \ln x(2y-1)^{-1} \end{aligned} \quad (33)$$

$$\overline{F}_C^\sigma - F_C^o = -F_{C+} + RT \ln (2y-1) 4^{-1} \alpha^{-2} y^{-1} \quad (34)$$

As pointed out previously, these equations are completely explicit only if  $W$  is known. Consequently  $W$  can be determined from limited data or approximated by  $W = 0$ . In the latter case, Eqs. 26 through 34 can be used to compute phase equilibria, vapor pressures, rates of free evaporation, or compositions of congruent vaporization.

### III. APPLICATION OF MODEL TO Ta - Hf - C SYSTEM

In order to apply the equations developed in Section II to the tantalum-hafnium-carbon system in explicit terms, it is necessary to assume that  $W = 0$ . Moreover since the NaCl type  $\bar{\sigma}$  phase exists for values of  $y$  (atomic fraction of carbon) which are less than or equal to 0.5, Eqs. 26 through 31 can be applied. In this system, tantalum is element A, hafnium is element B, and carbon is element C. Table I contains the relevant values for the free energy of formation of stoichiometric HfC and TaC. These values are the same as the values employed in an earlier study of the binary carbides.<sup>(2)</sup> Consequently, Eqs. (26) to (28) yield the following expressions for  $y = 0.5$ .

$$RT \ln p_{Ta}^{\bar{\sigma}} [x] / p_{Ta}^{\circ} = -F_{Ta} - 2x(23,100) - RT \ln \alpha + RT \ln (0.5-x) \quad (35)$$

$$RT \ln p_{Hf}^{\bar{\sigma}} [x] / p_{Hf}^{\circ} = -F_{Hf} + 2(0.5-x)(23,100) - RT \ln \alpha + RT \ln x \quad (36)$$

and

$$RT \ln p_C^{\bar{\sigma}} [x] / p_C^{\circ} = -F_C - RT \ln 2 \alpha \quad (37)$$

since  $1 - 2\alpha$  is nearly equal to unity. On the basis of Table I and Eqs. (9) and (10)

$$RT \ln \alpha = -35,900 - x(1-y)^{-1} 23,100 \text{ cal/g.at.} \quad (38)$$

Table I  
Summary of Numerical Values Required For  
Vapor Pressure Computations

$$2\Delta H_{TaC}^{\sigma}[0^{\circ}K] \approx -35,900 \text{ cal/mol}$$

$$2\Delta H_{HfC}^{\sigma}[0^{\circ}K] \approx -59,000 \text{ cal/mol}$$

| $T^{\circ}K$ | $2\Delta F_{TaC}^{\sigma}[T]$<br>cal/mol | $F_{Ta+}$<br>cal/g.at. | $RT \ln 2$<br>cal/g.at. | $2\Delta F_{HfC}^{\sigma}[T]$<br>cal/mol | $F_{Hf+}^{\sigma}$<br>cal/g.at. |
|--------------|--|------------------------|-------------------------|--|---------------------------------|
| 2200         | -33,260                                  | +66,126                | 3034                    | -54,200                                  | 110,165                         |
| 2600         | -32,780                                  | +65,095                | 3585                    | -53,100                                  | 108,515                         |
| 2800         | -32,540                                  | +64,579                | 3861                    | -52,100                                  | 107,240                         |
| 3000         | -32,300                                  | +64,063                | 4137                    | -51,200                                  | 106,065                         |

source: Ref(2) pp(21) and (69)

| $T^{\circ}K$ | $-\log p_C^{\circ}$                | $-\log p_{Ta}^{\circ}$ | $-\log p_{Hf}^{\circ}$ |
|--------------|------------------------------------|------------------------|------------------------|
|              | (pressure in units of atmospheres) |                        |                        |
| 2200         | 8.781                              | 11.206                 | 7.382                  |
| 2600         | 6.174                              | 8.346                  | 5.246                  |
| 2800         | 5.151                              | 7.219                  | 4.412                  |
| 3000         | 4.266                              | 6.243                  | 3.692                  |

source: C - JANAF Thermochemical Tables

Hf and Ta - S.A.R. #2 AF33(657)8635 - "Borides" p VII f-6

$$RT \ln \alpha = -35,900 - x(1-y)^{-1} 23,100 \text{ (from Eqs. (9), (10))}$$

$$F_{C+}[x, T] = -RT \ln 2 \alpha \quad \text{(from Eq. (9))}$$

$$(F_{Hf+} - F_{Ta+} + 2(\Delta F_{HfC}^{\sigma} - \Delta F_{TaC}^{\sigma})) = 23,100 \text{ cal/g.at. (from Eqs. (11) and (12))}$$

When  $y < 0.5$ , Eqs. 29 to 31 yield

$$RT \ln p_{Ta}^{\sigma} [x, y] / p_{Ta}^{\circ} = -F_{Ta+} - x(1-y)^{-1} 23,100 - 2RT \ln \alpha \\ + RT \ln (1-x-y) (1-2y)^{-1} (1-y)^{-2} \quad (39)$$

$$RT \ln p_{Hf}^{\sigma} [x, y] / p_{Hf}^{\circ} = -F_{Hf+} + (1-x-y) (1-y)^{-1} 23,100 - 2RT \ln \alpha \\ + RT \ln x (1-2y)^{-1} (1-y)^{-2} \quad (40)$$

and

$$RT \ln p_C^{\sigma} [x, y] / p_C^{\circ} = -F_{C+} + RT \ln y (1-2y)^{-1} \quad (41)$$

Eqs. 35 through 41 define the vapor pressures of tantalum, hafnium, and carbon as a function of  $x$ ,  $y$  and  $T$ . Numerical calculation of these quantities has been performed for four carbon levels,  $y = 0.50, 0.49, 0.46$ , and  $0.41$  at temperatures of  $2200, 2600, 2800$  and  $3000^{\circ}\text{K}$ . Graphical representations of these results are shown in Figs. 1 to 16.\* In addition, the rates of Langmuir evaporation for each component has been computed as follows:

$$\dot{v}_{Ta} = 44.4 p_{Ta}^{\sigma} [x, y, T] (M_{Ta})^{1/2} T^{-1/2} \text{ gms/cm}^2 \text{ sec} \quad (42)$$

$$\dot{v}_{Hf} = 44.4 p_{Hf}^{\sigma} [x, y, T] (M_{Hf})^{1/2} T^{-1/2} \text{ gms/cm}^2 \text{ sec} \quad (43)$$

$$\dot{v}_C = 44.4 p_C^{\sigma} [x, y, T] (M_C)^{1/2} T^{-1/2} \text{ gms/cm}^2 \text{ sec} \quad (44)$$

where  $M_{Ta}$ ,  $M_{Hf}$ , and  $M_C$  are the atomic masses of tantalum, hafnium, and carbon respectively.

\* These temperatures and compositions were agreed upon at a meeting between L. Kaufman, of ManLabs and H.B. Probst, D. Deadmore and I. Zaplatynsky of N.A.S.A.



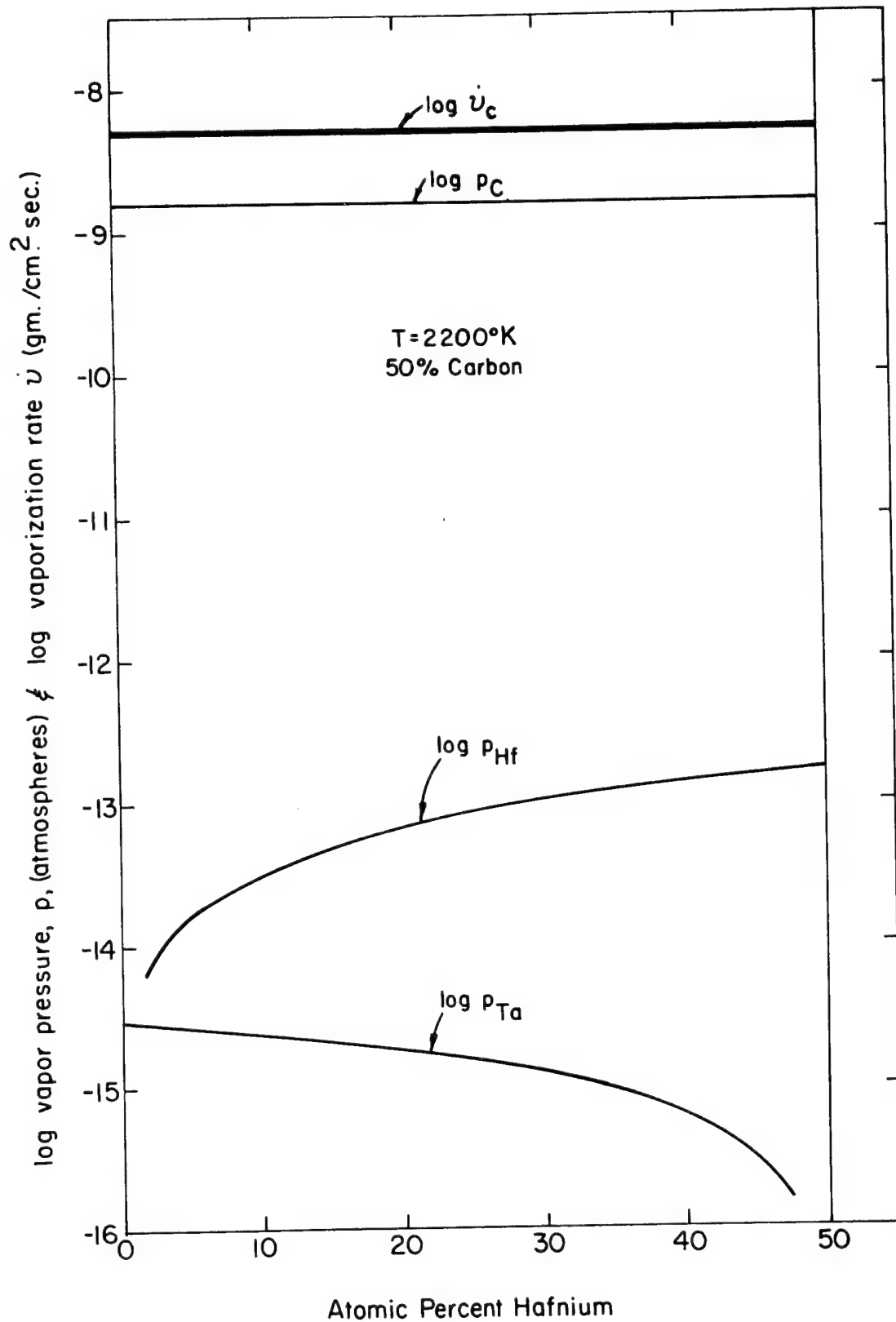
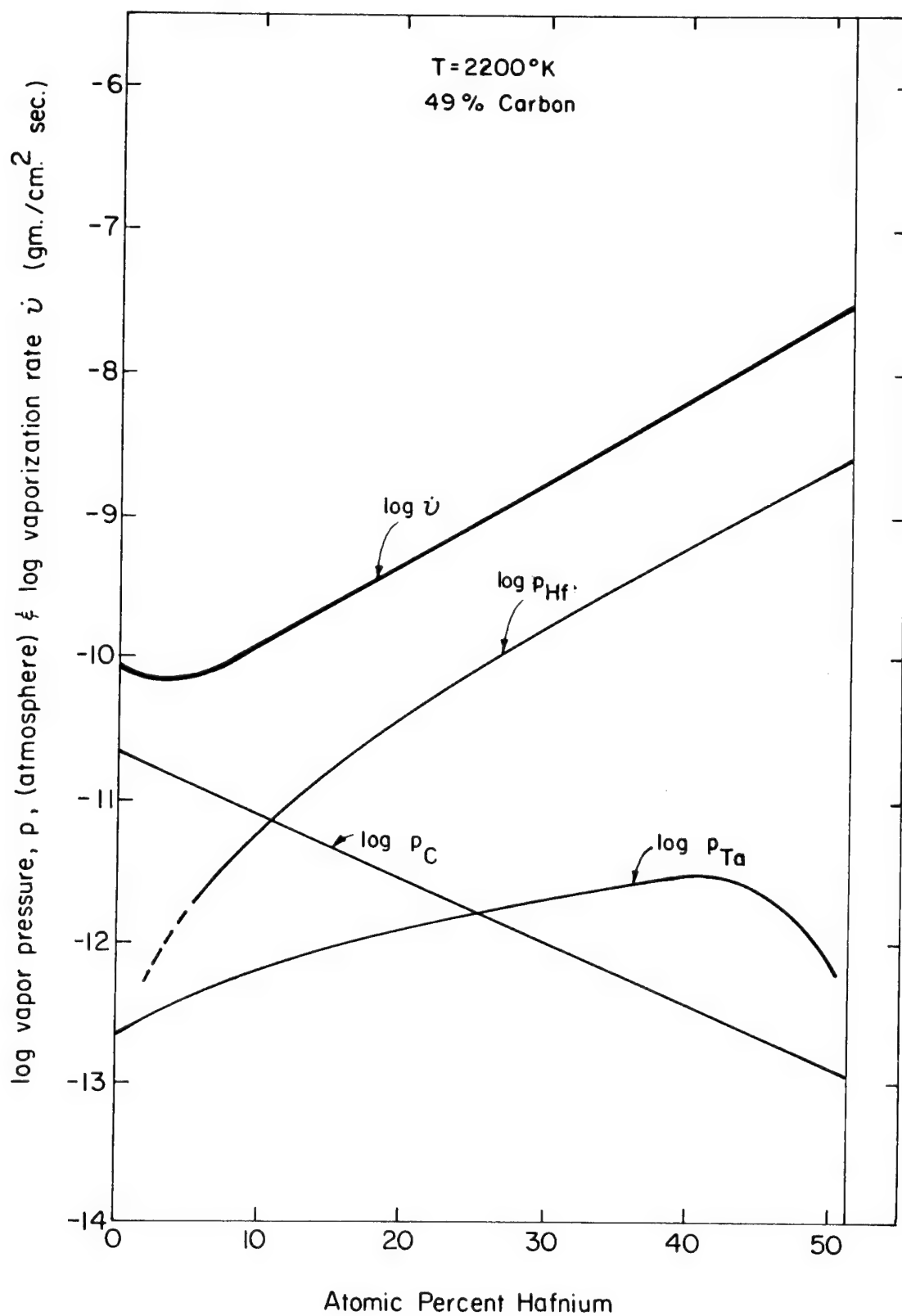


Fig. 1 - Vaporization of (Ta-Hf)C at 2200°K and 50% carbon.



**Fig. 2** - Vaporization of (Ta-Hf)C at 2200°K and 49% carbon.

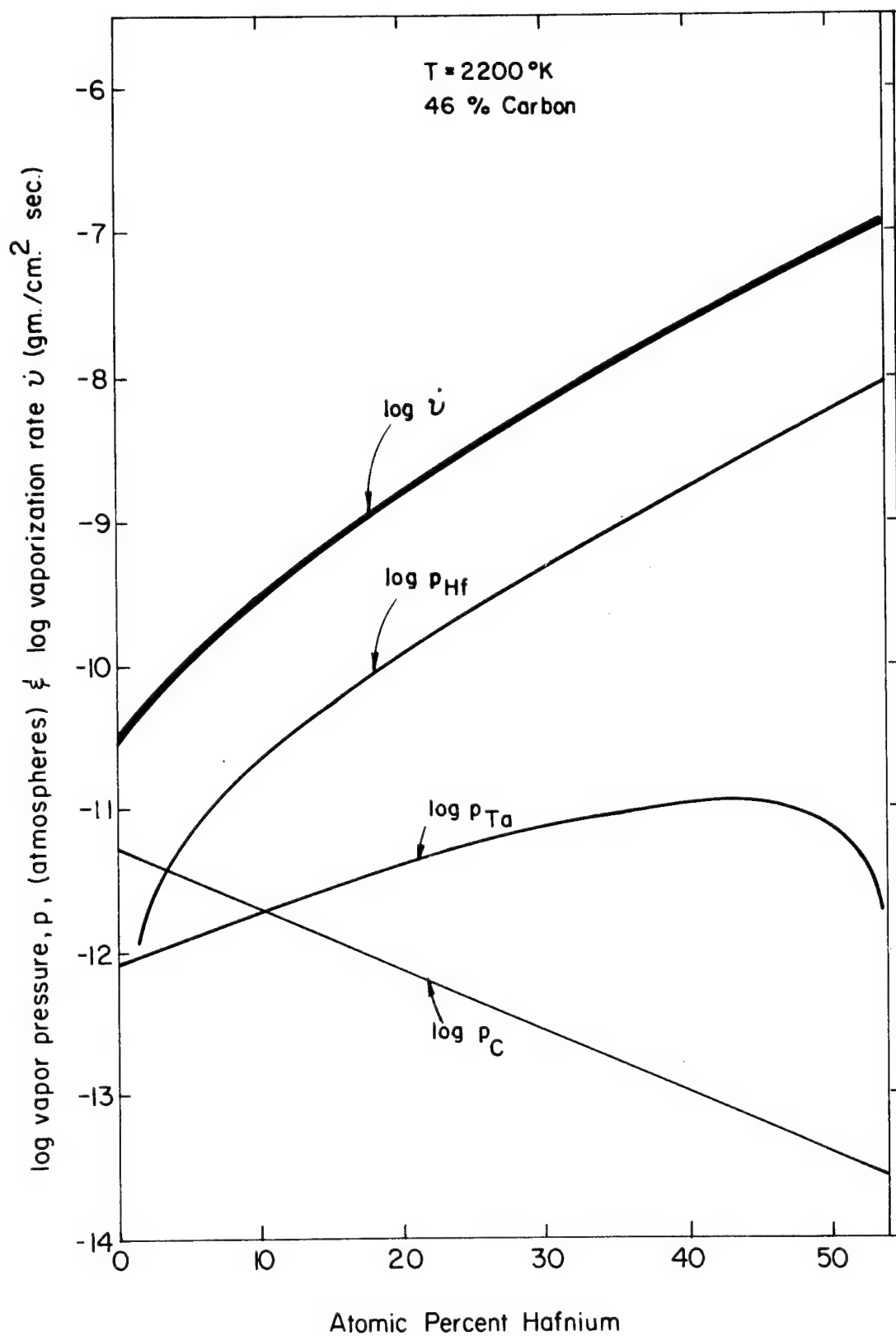


Fig. 3 - Vaporization of (Ta-Hf)C at 2200°K and 46% carbon.

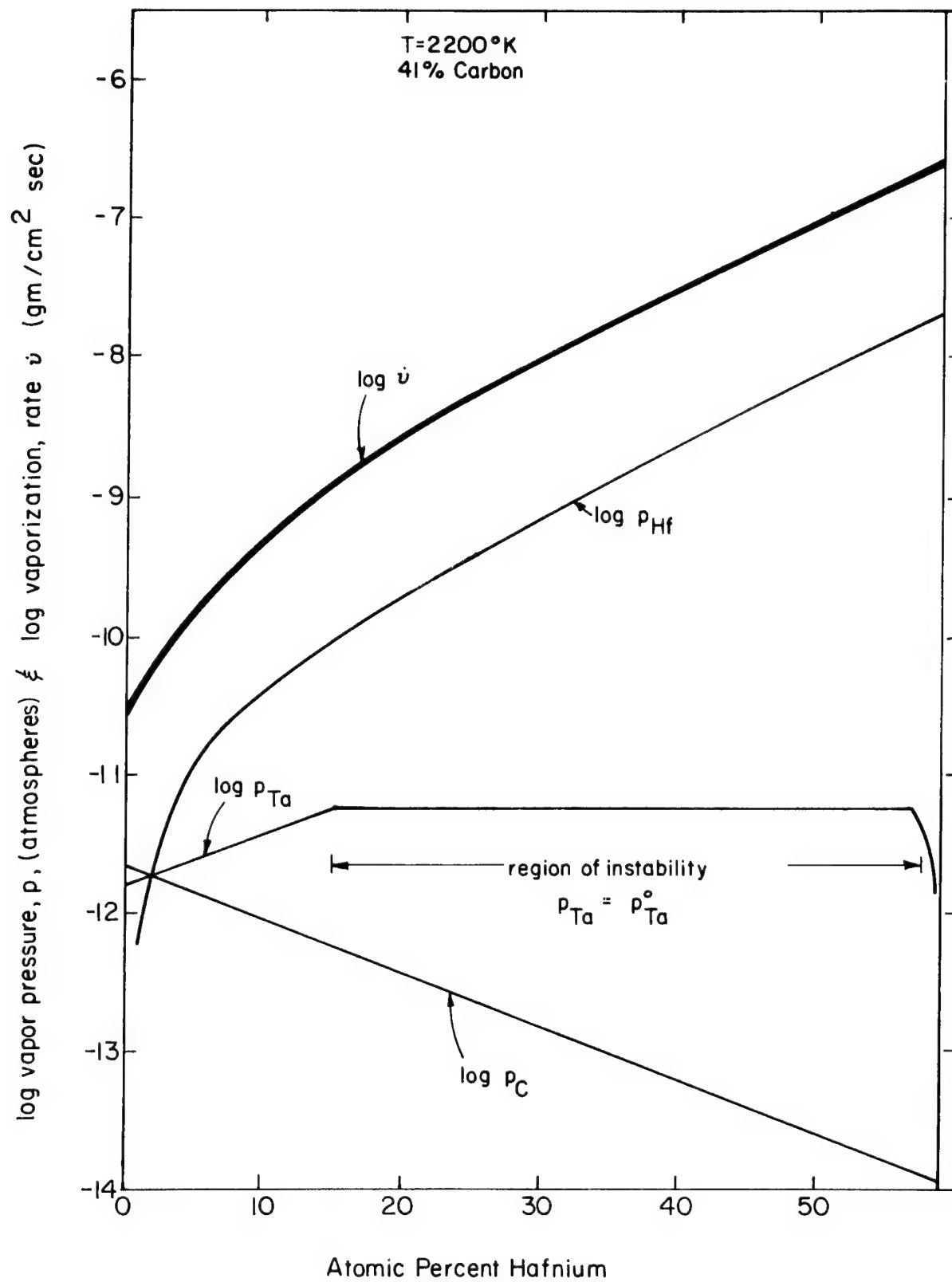


Fig. 4 - Vaporization of (Ta-Hf)C at 2200°K and 41% Carbon.

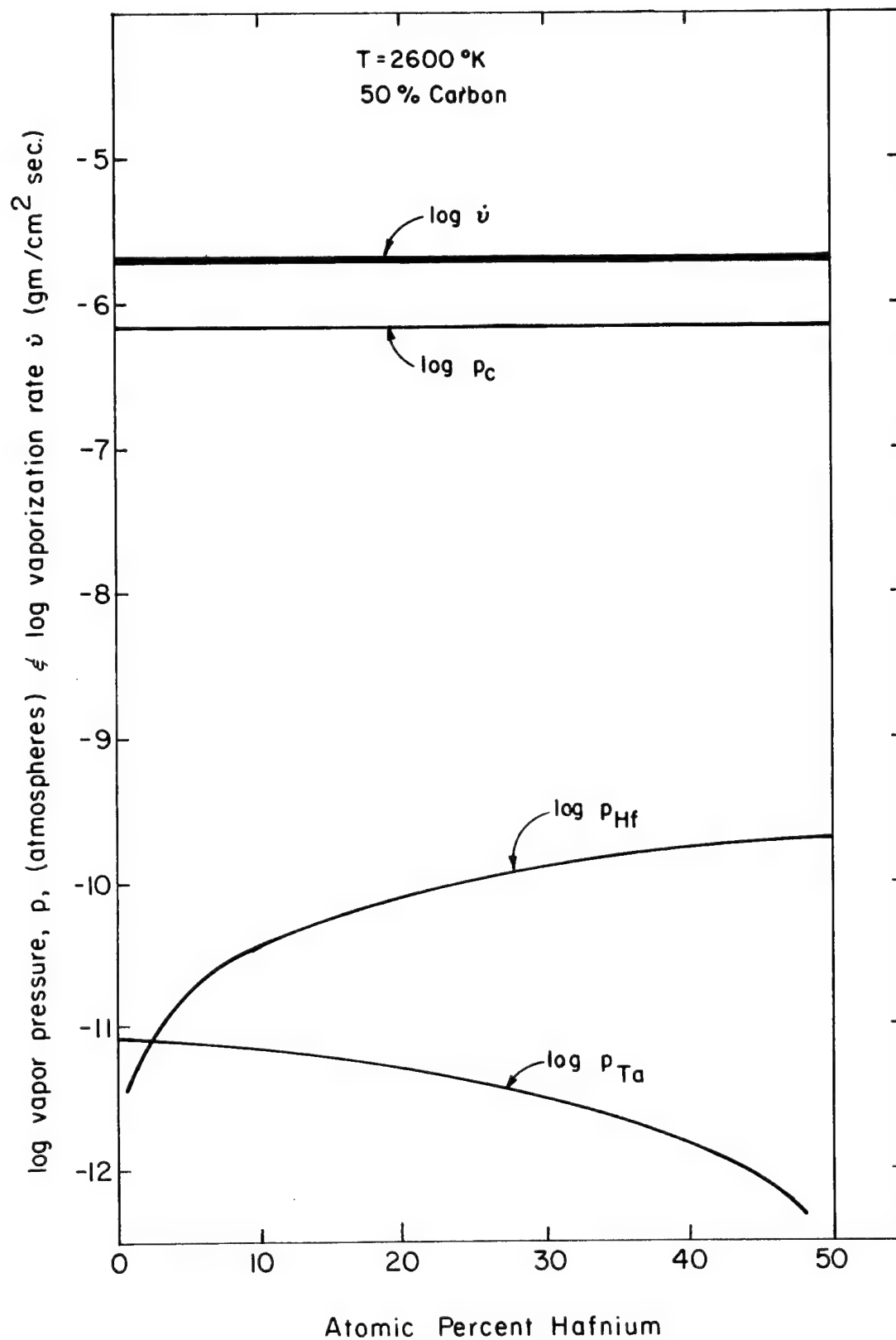


Fig. 5 - Vaporization of (Ta-Hf) C at 2600 °K and 50% carbo

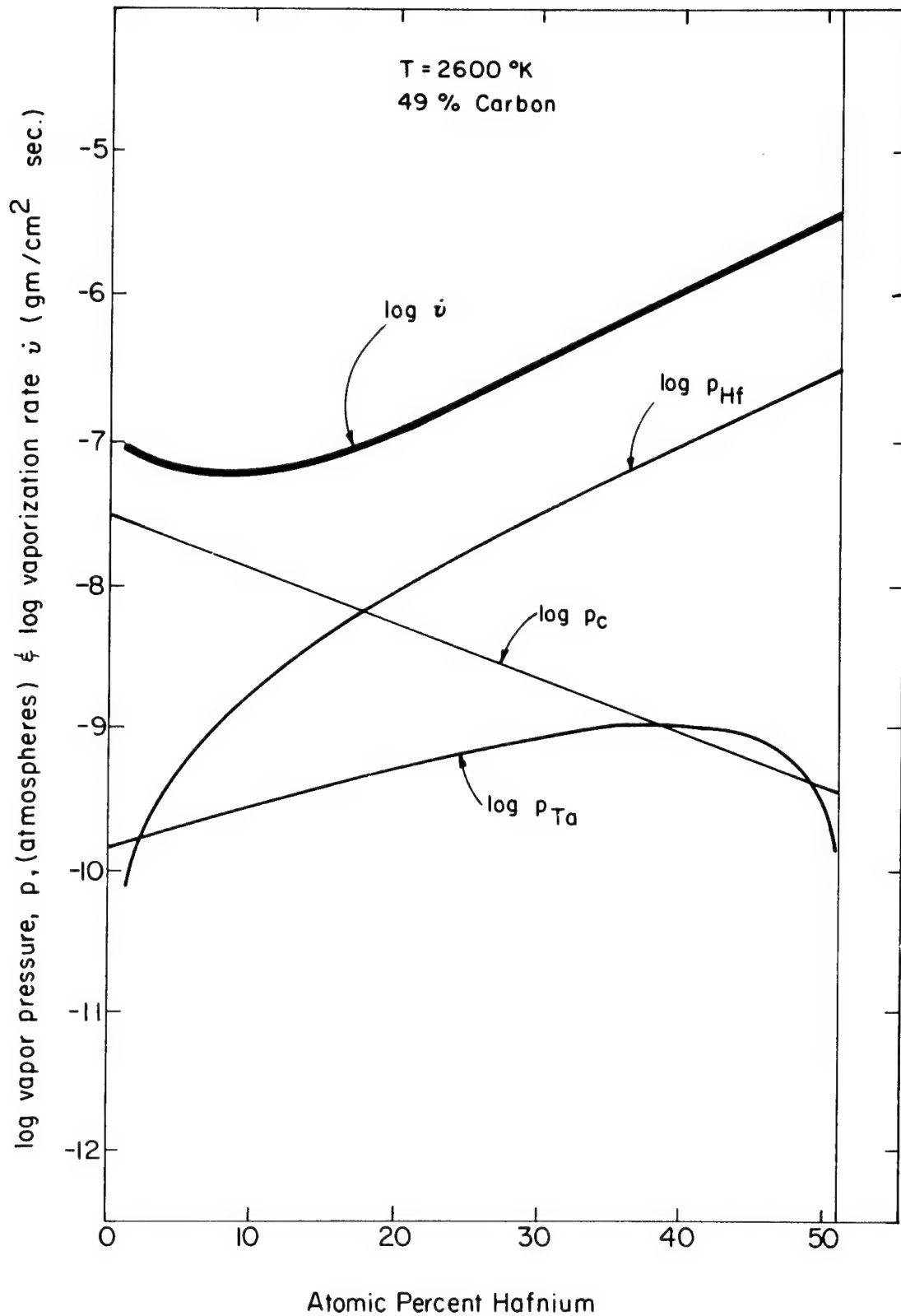


Fig. 6 - Vaporization of (Ta-Hf) C at  $2600^\circ\text{K}$  and 49% carbon.

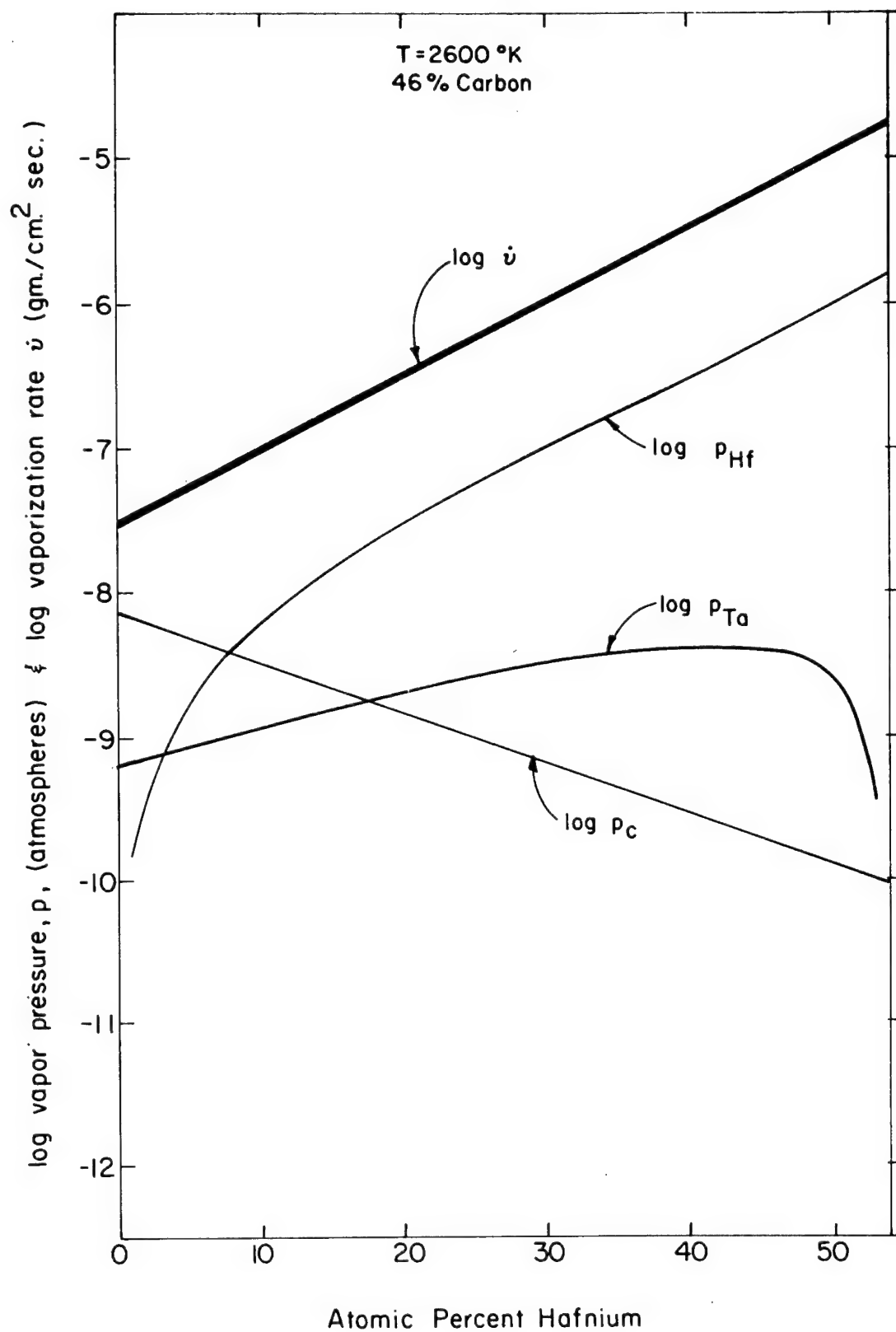


Fig. 7 - Vaporization of (Ta-Hf)C at 2600°K and 46% carbon.

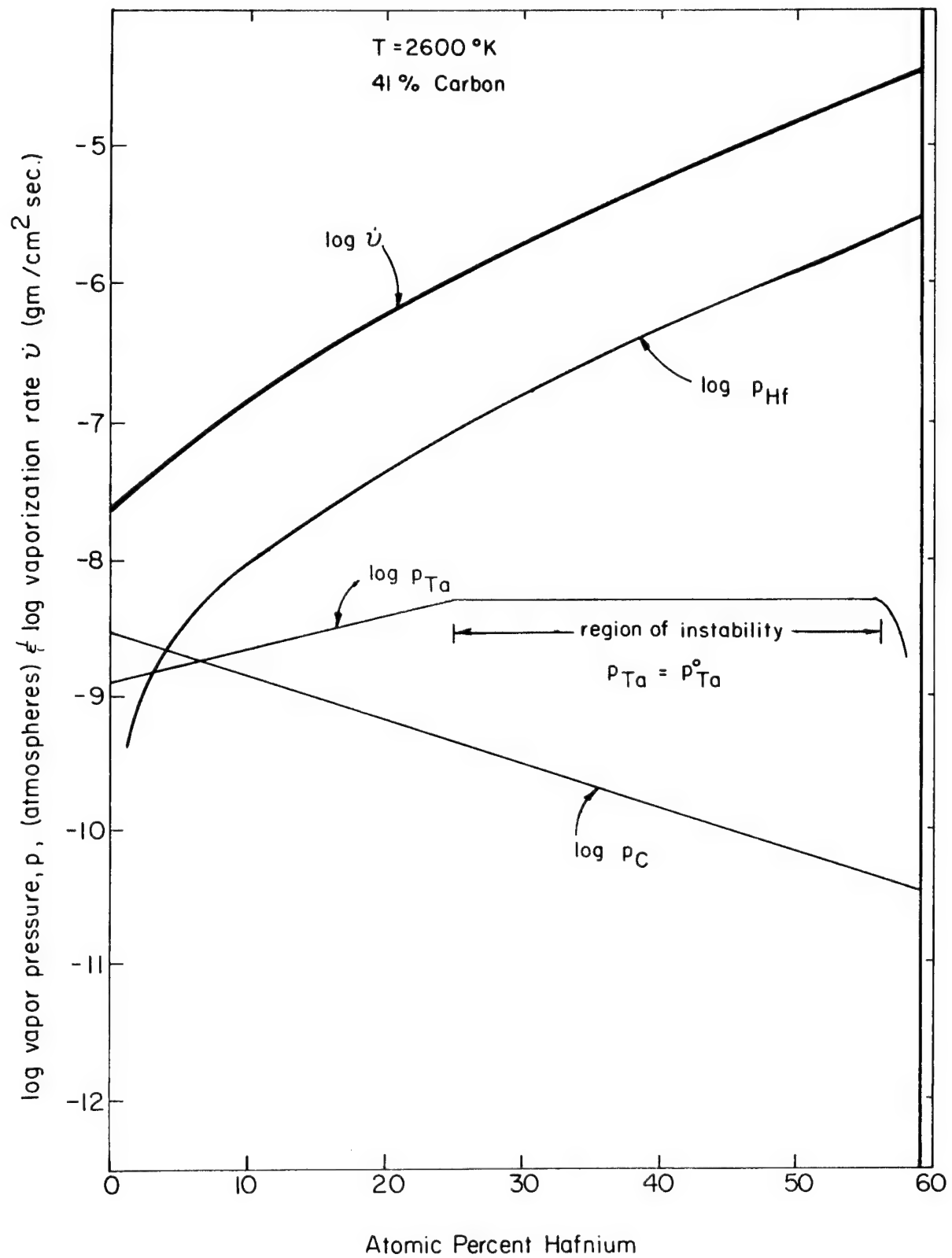


Fig. 8 - Vaporization of (Ta-Hf)C at 2600°K and 41% carbon.



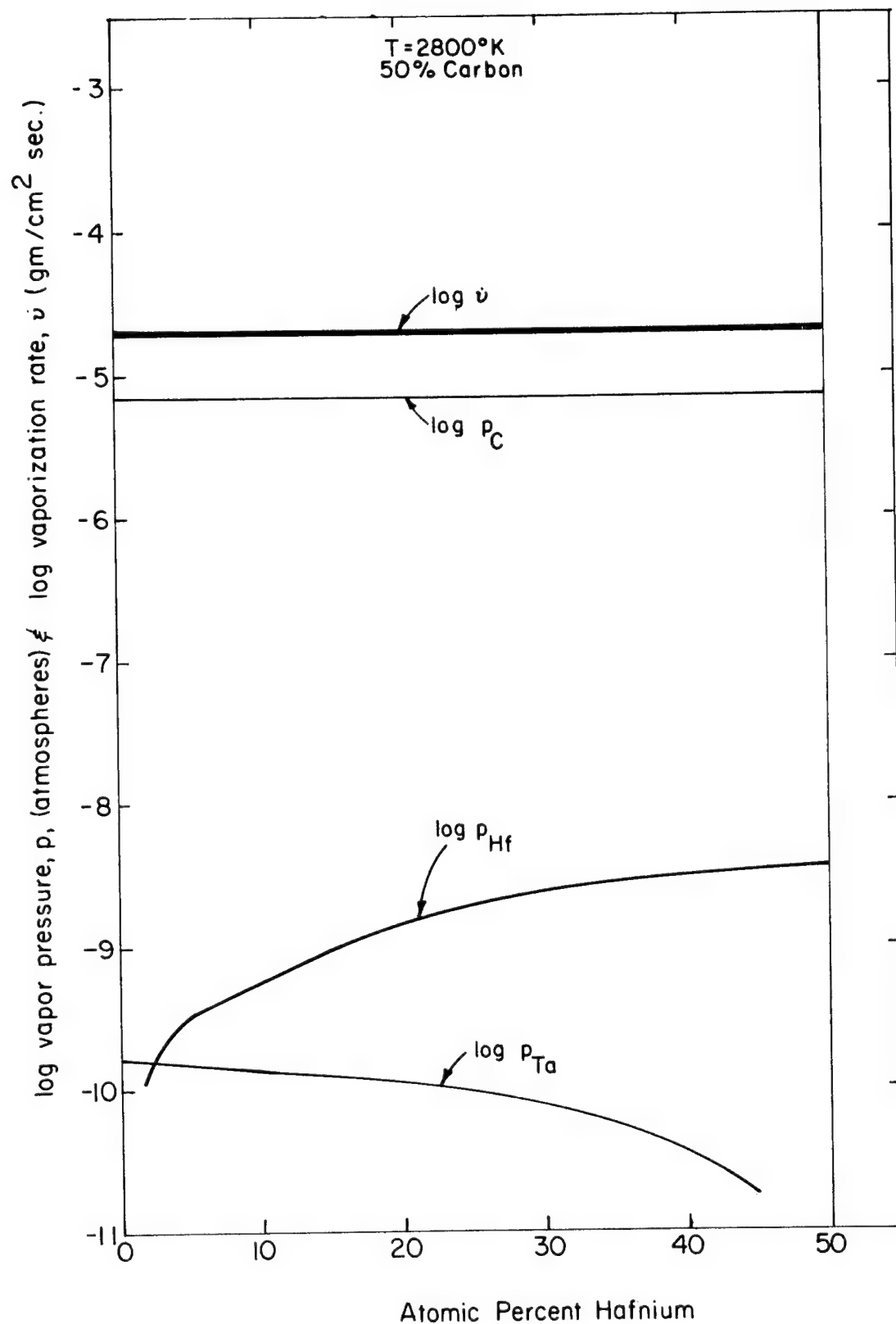


Fig. 9 - Vaporization of (Ta-Hf) C at  $2800^{\circ}\text{K}$  and 50% carbon.

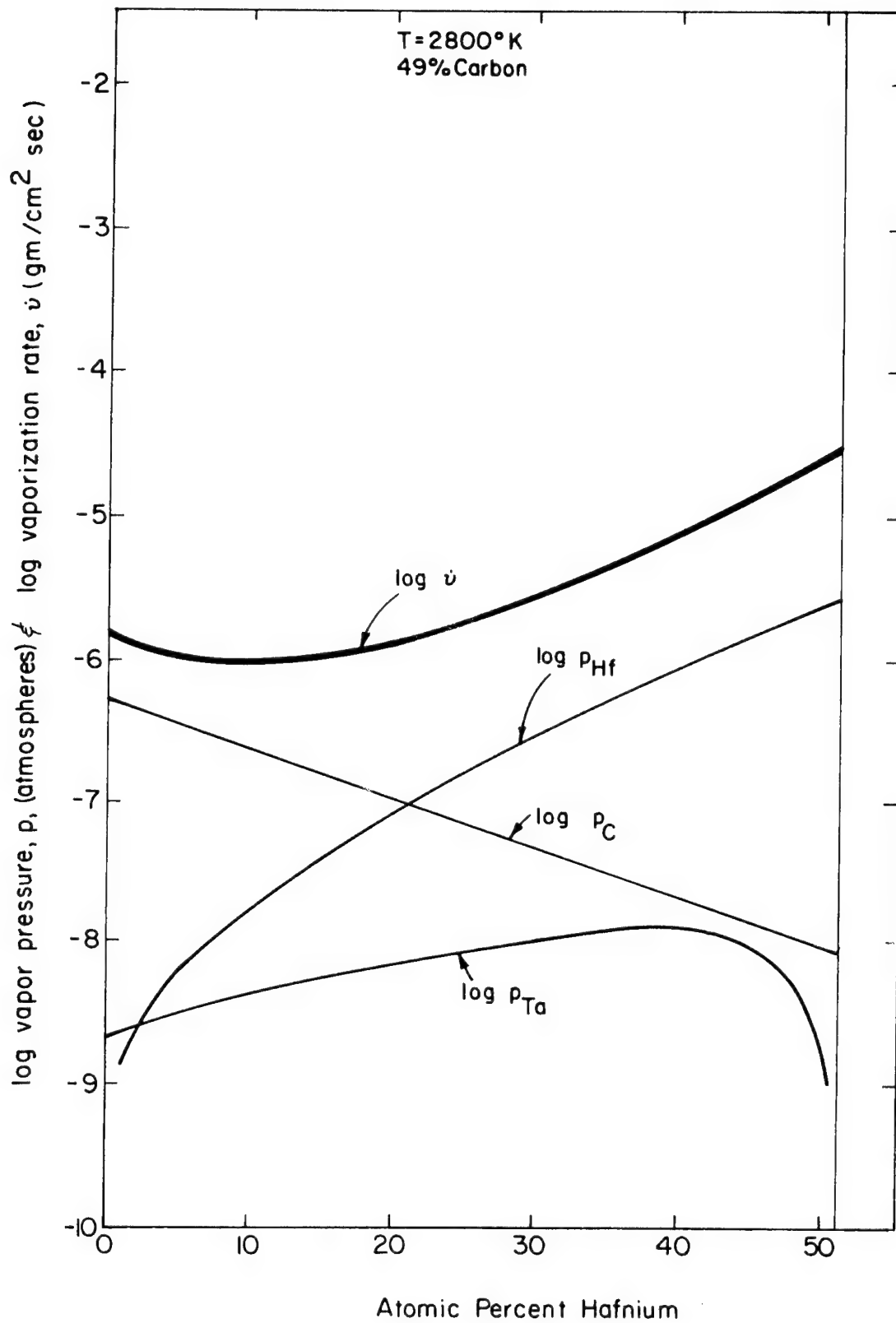


Fig. 10 - Vaporization of (Ta-Hf) C at 2800°K and 49% carbon.

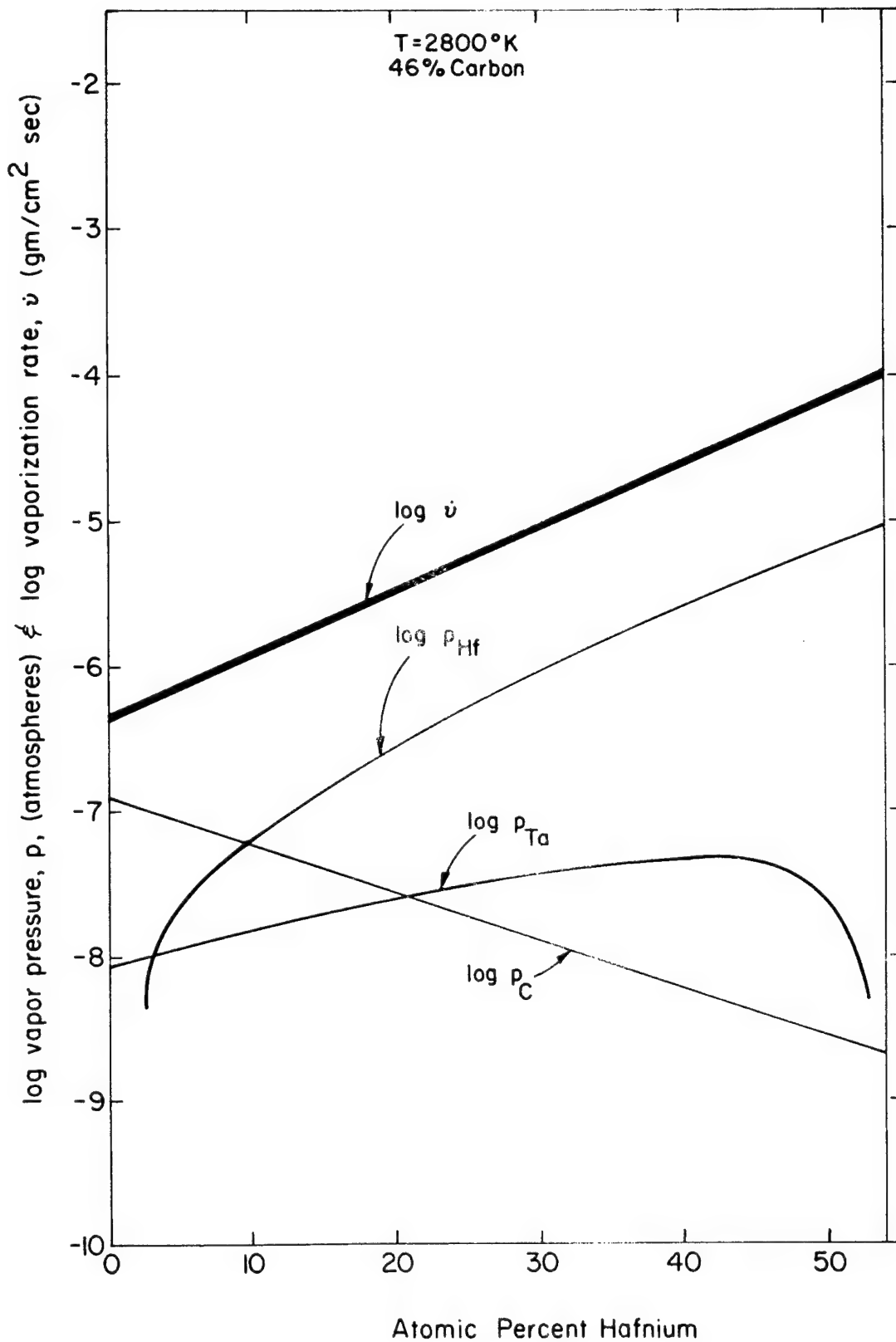


Fig. 11 - Vaporization of (Ta-Hf) C at 2800°K and 46% carbon.

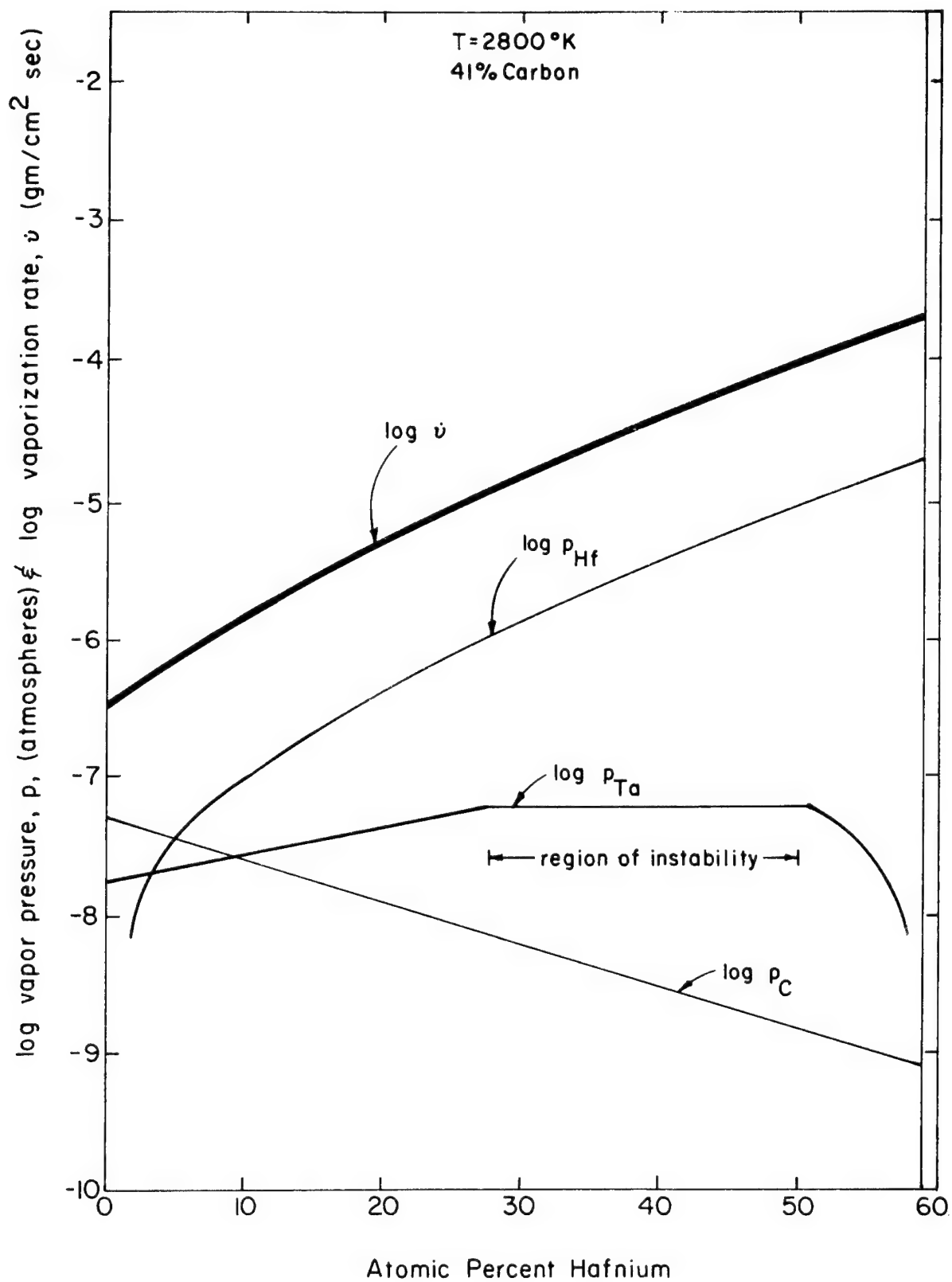
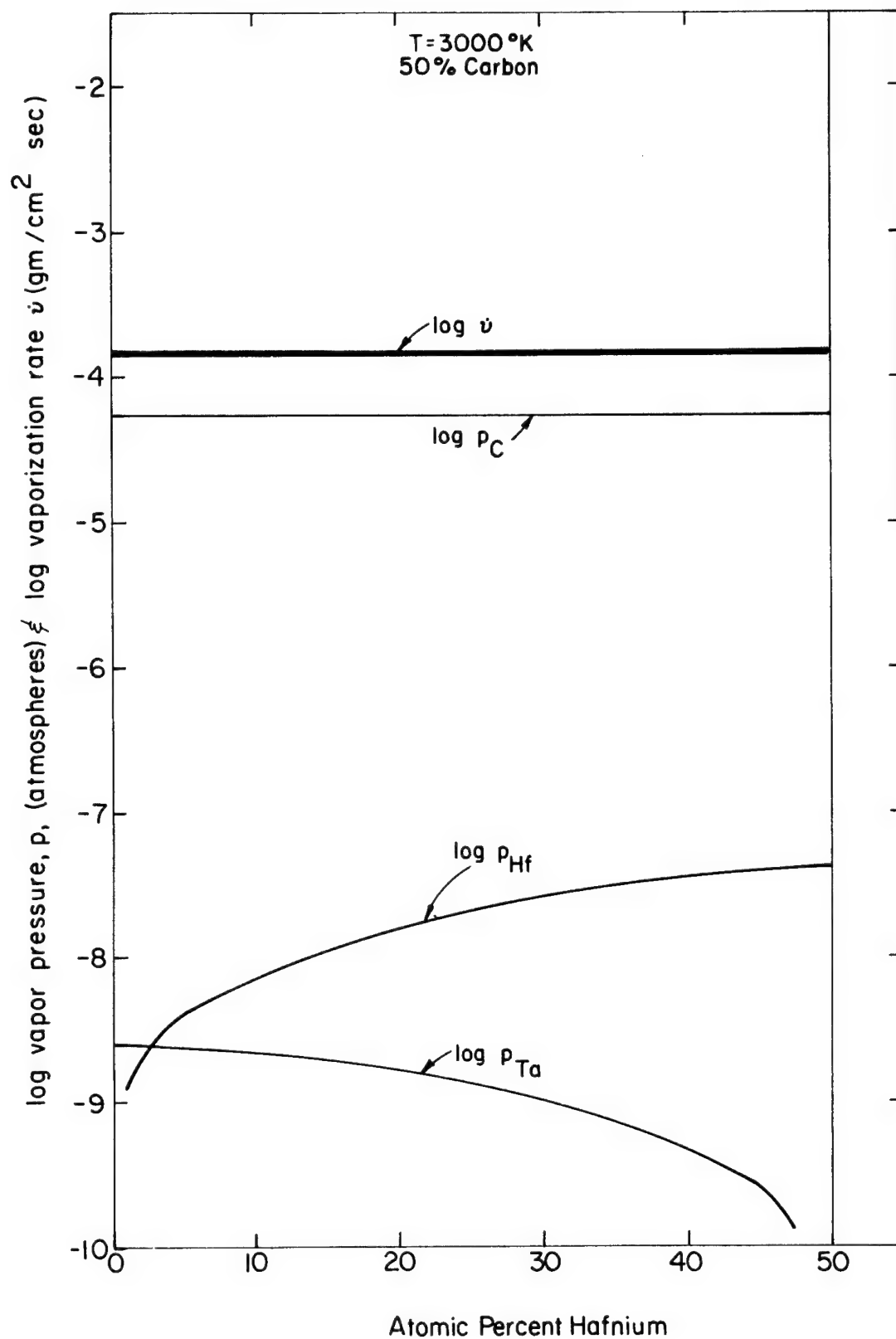


Fig. 12 - Vaporization of (Ta-Hf) C at  $2800^{\circ}\text{K}$  and 41% carbon.



**Fig. 13** - Vaporization of (Ta-Hf)C at 3000°K and 50% carbon.

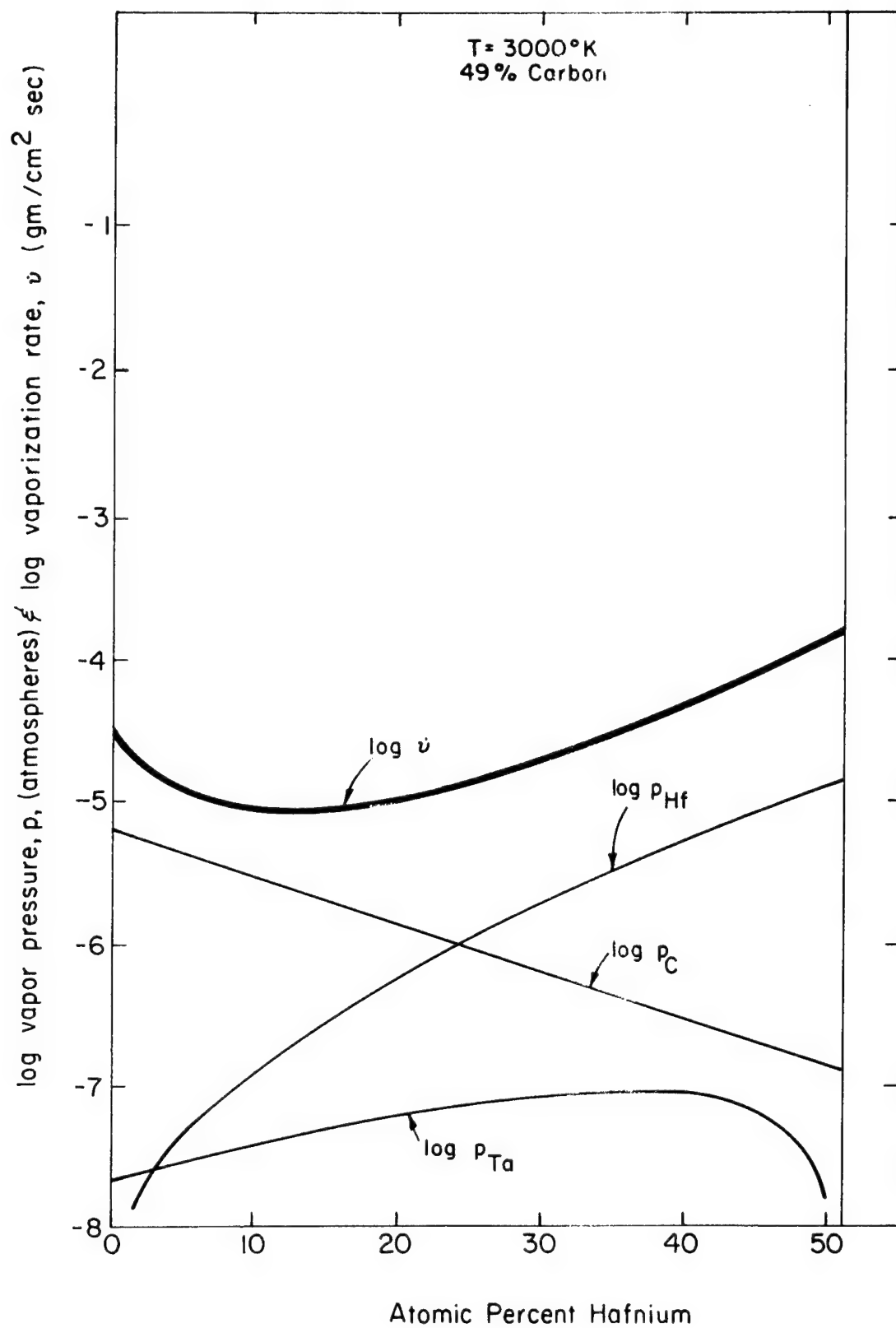


Fig. 14 - Vaporization of (Ta-Hf)C at 3000°K and 49% carbon.

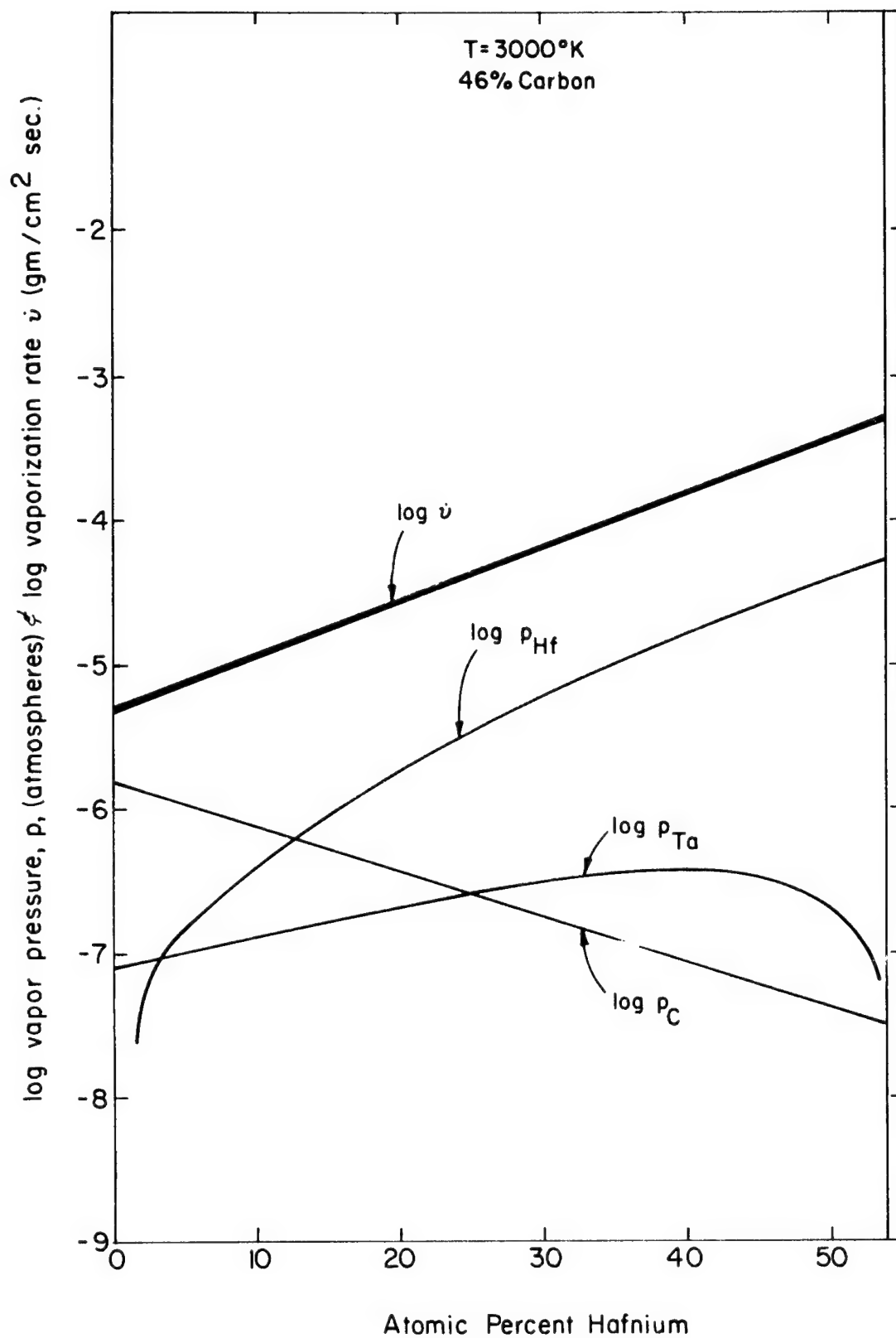


Fig. 15 - Vaporization of (Ta-Hf) C at 3000°K and 46% carbon.

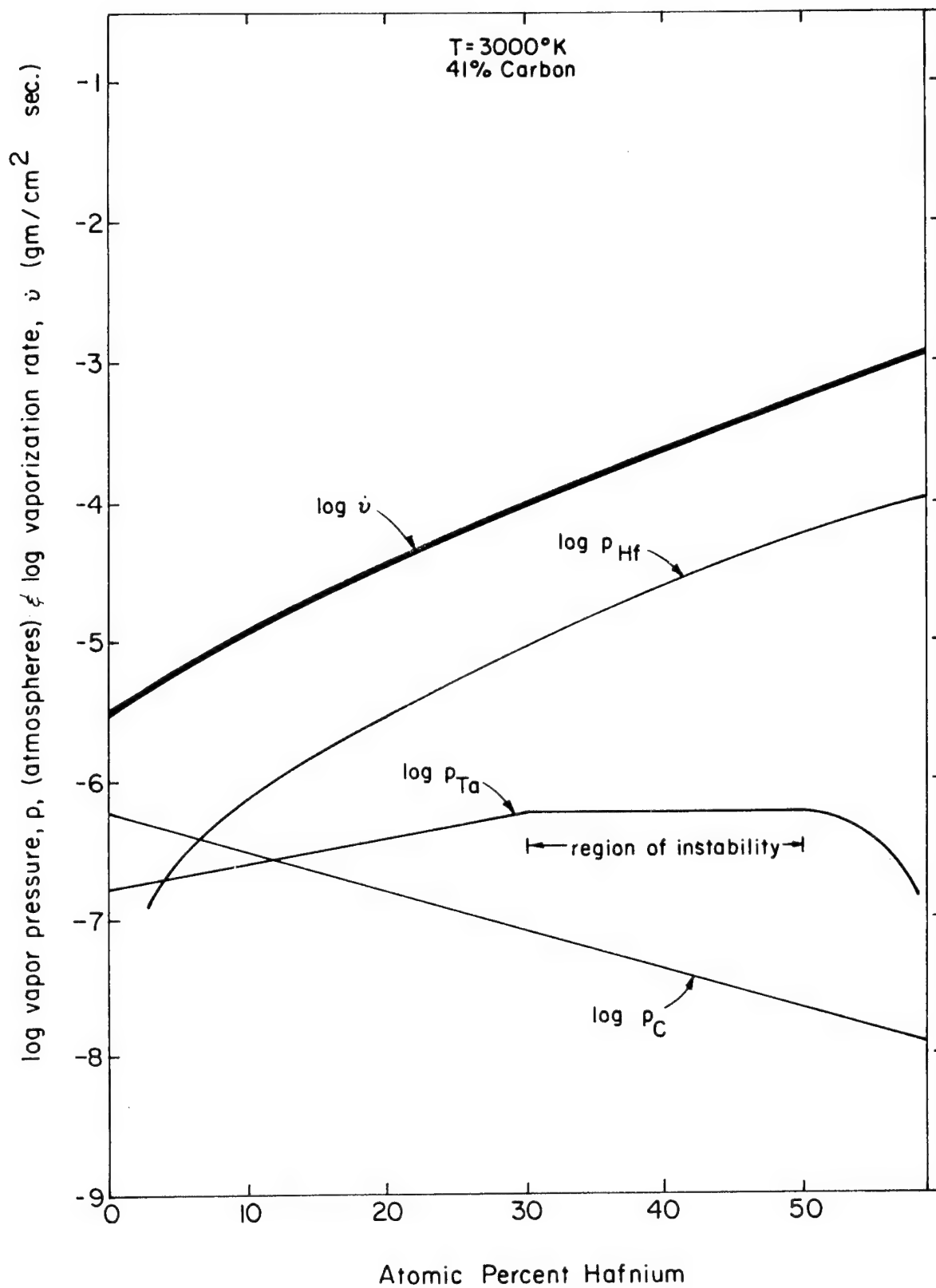


Fig. 16 - Vaporization of (Ta-Hf)C at 3000°K and 41% carbon.



The total rate of vaporization  $\dot{v}$  has been computed by summation of the individual rates i.e.

$$\dot{v} = \dot{v}_{Ta} + \dot{v}_{Hf} + \dot{v}_C \quad (45)$$

Figs. (1-16) show  $\dot{v}$  [x, y, T] in addition to the individual pressures.

Reference to Figs. 1-16 shows that for most of the composition and temperature ranges explored, the vapor pressure of Ta is substantially less than Hf or C. Thus, all of the compositions will lose hafnium and carbon relative to tantalum. Consequently the real rates of vaporization should change with time reflecting changes in the composition. The computed values of  $\dot{v}$  correspond to fixed compositions.

In the Hf-C binary, congruency phenomena are predicted and have been observed (see pp 21-38 of reference 2). These conditions correspond to unique cases where the rates of carbon and hafnium vaporization are such, that the ratio of carbon to hafnium atoms leaving the solid is the same as the ratio of carbon to hafnium atoms in the solid. This condition is represented explicitly by Eq. (46). \*

$$(\dot{v}_C [x_c, y_c, T] / \dot{v}_{Hf} [x_c, y_c, T]) = y_c / x_c = \frac{P_C^\sigma [x_c, y_c, T] M_{Hf}^{1/2}}{P_{Hf}^\sigma [x_c, y_c, T] M_C^{1/2}} \quad (46)$$

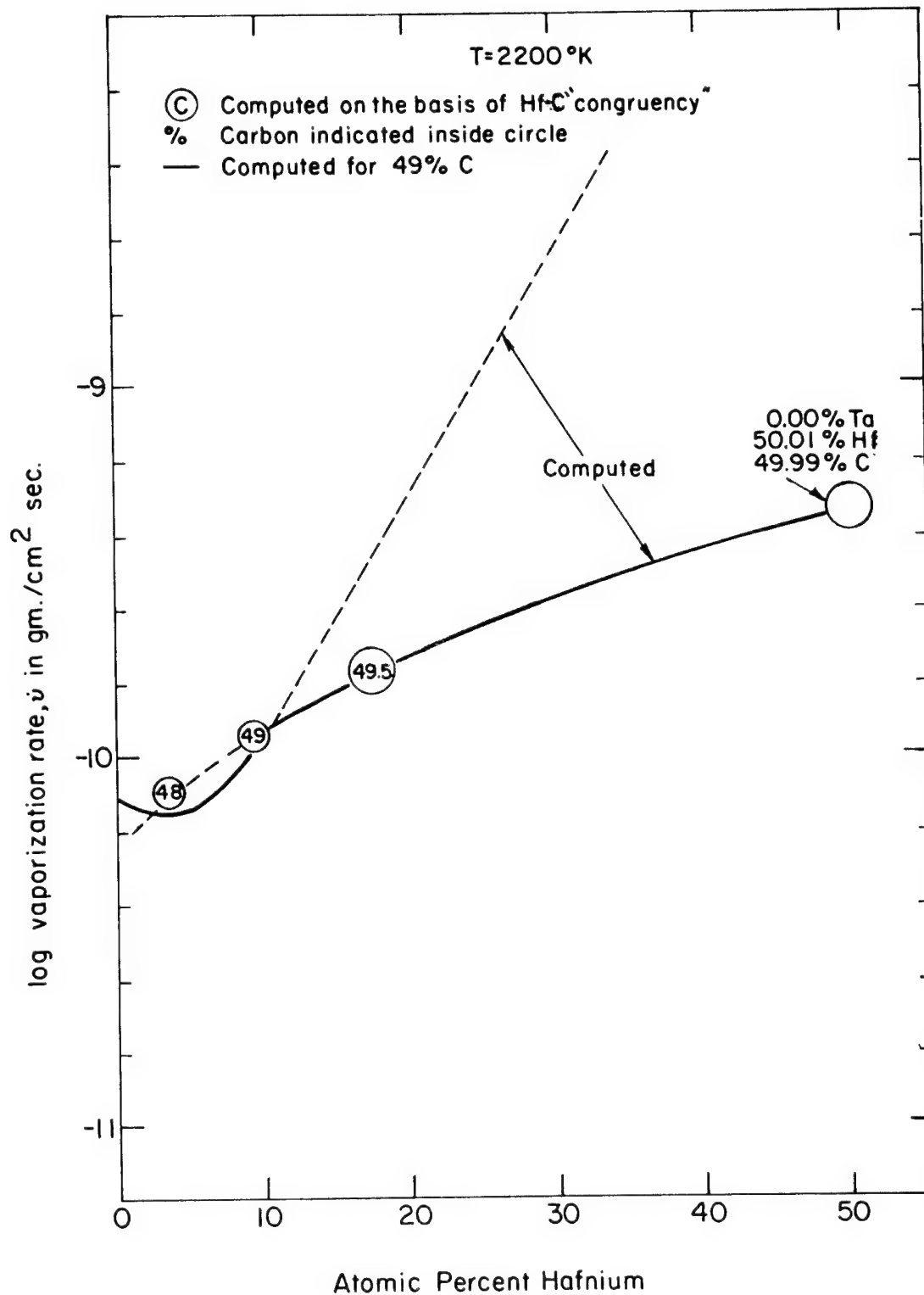
\*In Eq. 46, the vaporization rates are in moles/cm<sup>2</sup> sec.

when  $y = 1-x$ , i.e. no tantalum is present, the establishment of congruency yields a stable composition. This situation is shown graphically in Figs. (10) and (11) of ref (2) and compared with experiment in Figs. (18) and (19) of ref (2). Thus the computed rate of vaporization for a congruency situation should be stable since the bulk composition does not change with time.

Since there are no such ternary congruency points indicated for Ta-Hf-C alloys (i.e. no solutions for Eq. (46) appear except when  $y_c = 1-x_c$ ), it is expected that all of the compositions and rates of vaporization will be time dependent. However, it is possible to compute "quasi-congruency" conditions corresponding to situations where the ratio of Hf to C leaving the solid is in the same ratio as hafnium atoms to carbon atoms in the solid. This computation is performed by applying Eq. (46) to Eqs. 40 and 41 yielding,

$$\begin{aligned}
 2RT \ln (1-2y_c) = & RT \ln p_C^0 / p_{Hf}^0 + 0.5 RT \ln M_{Hf} / M_C \\
 & + F_{Hf} + 3RT \ln \alpha - (1-x_c - y_c) (1-y_c)^{-1} 23,100 \\
 & + RT \ln 8 (1-y_c)^2
 \end{aligned} \tag{47}$$

Eq. (47) can be employed by substituting various values for  $y_c$ , (i.e.  $y_c = 1-x_c$ ,  $y_c = 0.495$ ,  $y_c = 0.49$ ,  $0.48$  etc.) and obtaining the corresponding values of  $x_c$ . These companion values of  $x_c$  and  $y_c$  can then be substituted into Eqs. 39 to 45 to compute  $\dot{V} [x_c, y_c, T]$ . Figs. 17 to 20 show the results of this computation.



Computed initial rates of vaporization at  $2200^\circ\text{K}$ .

Fig. 17 - Computed Initial Rates of Vaporization at  $2200^\circ\text{K}$ .

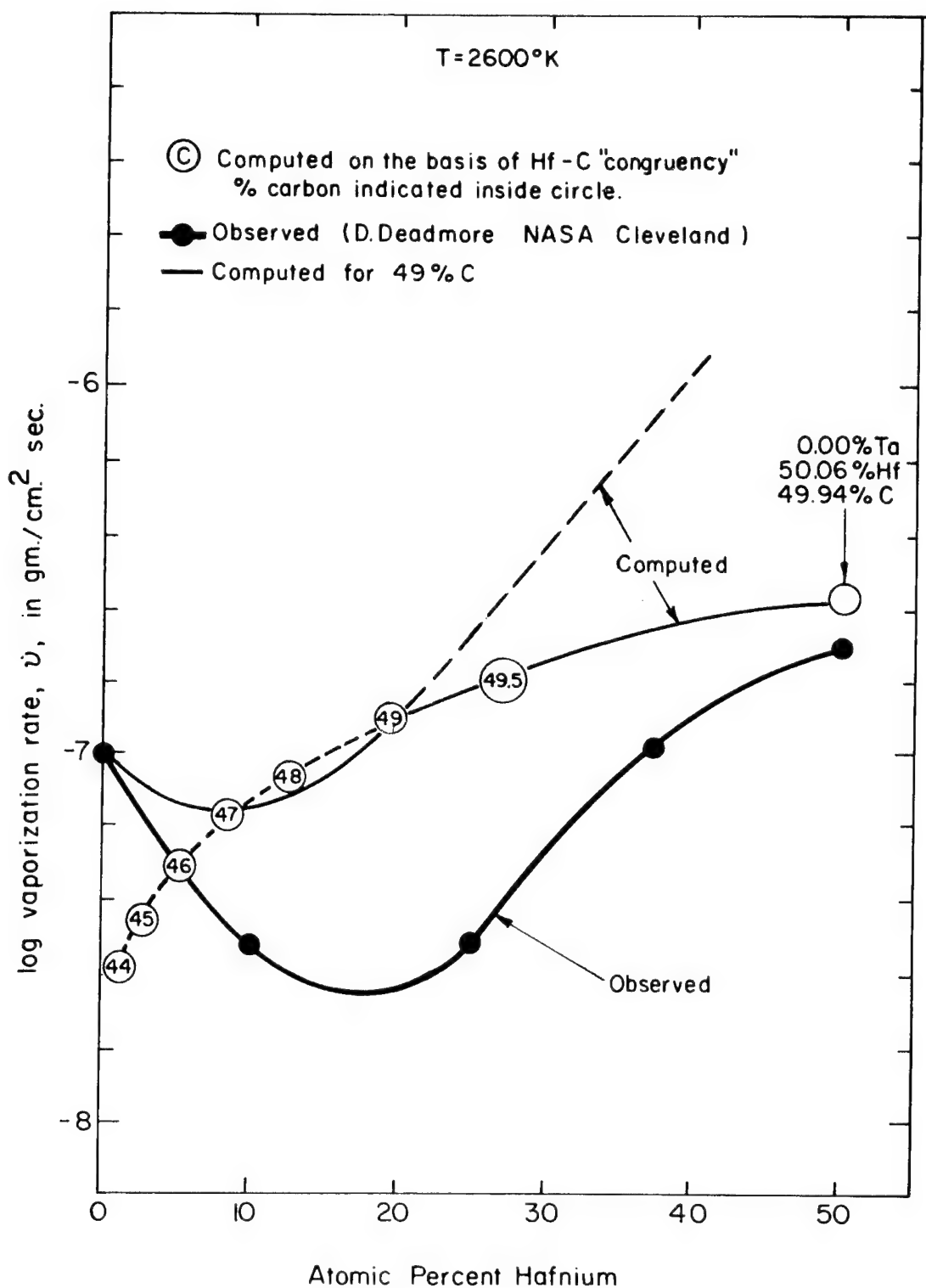
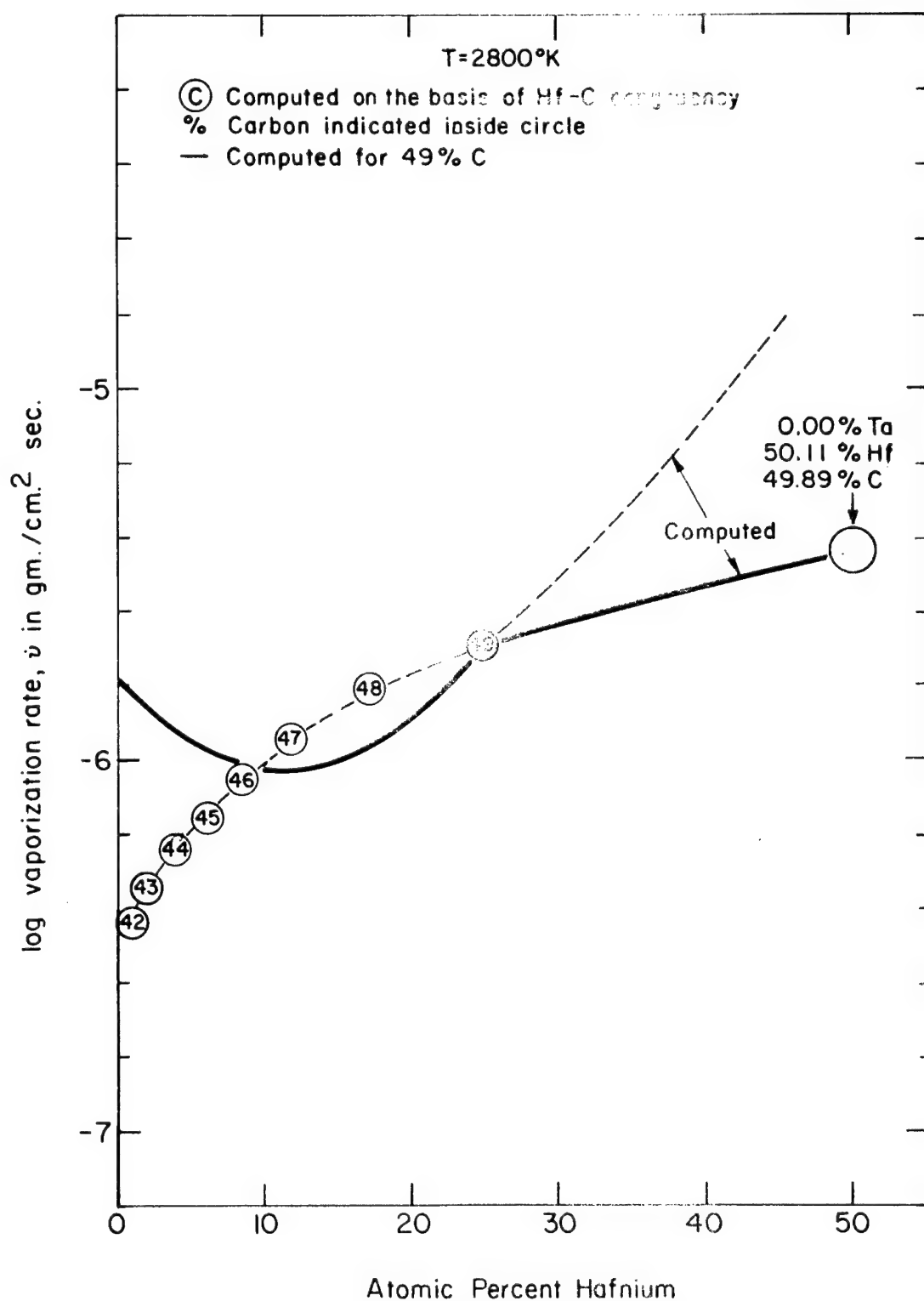
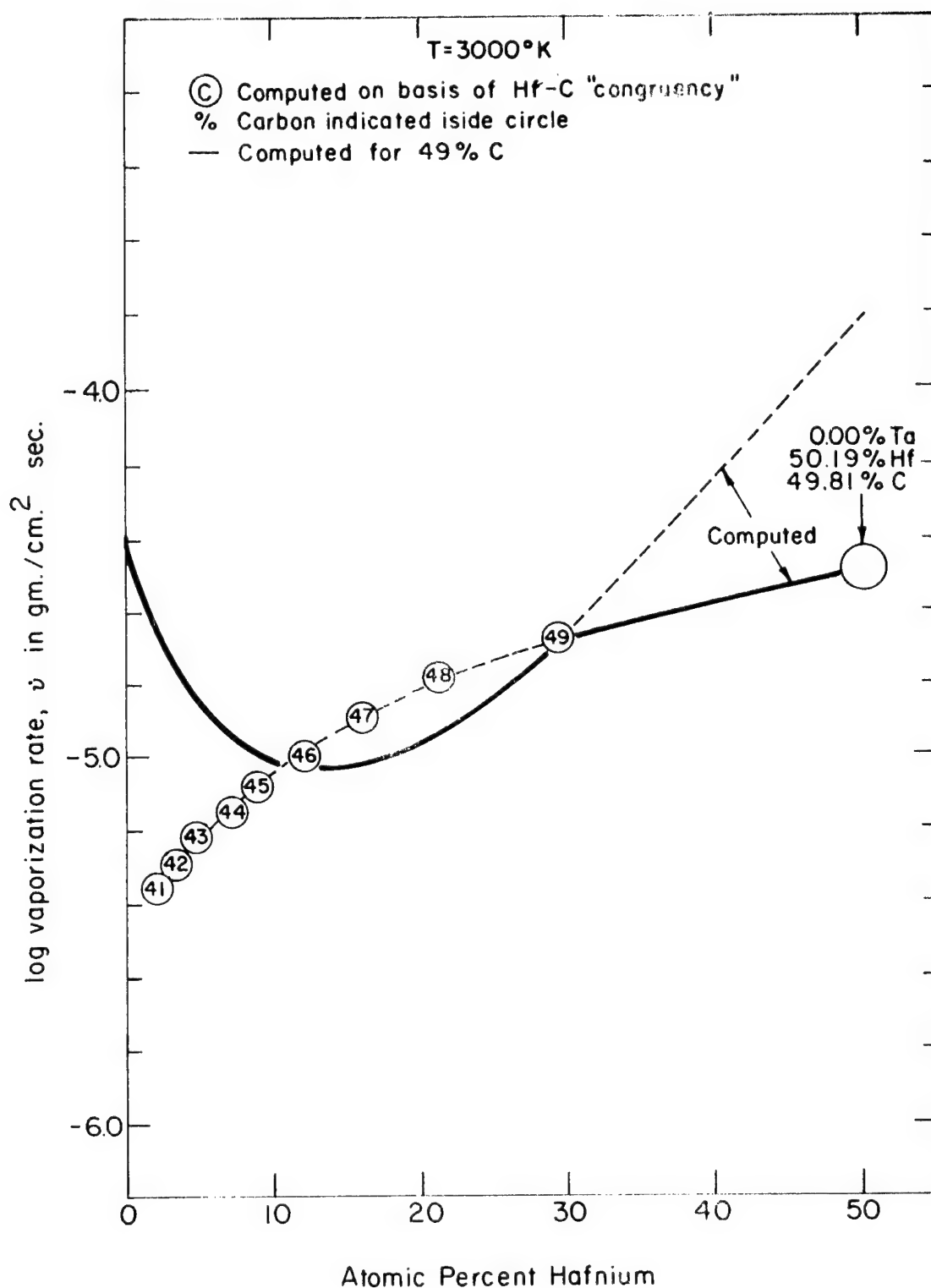


Fig. 18 - Comparison of observed (average) rates of vaporization with computed (initial) rates of vaporization at  $2600^{\circ}\text{K}$ .



Computed initial rate of vaporization at  $2800^{\circ}\text{K}$ .

Fig. 19 - Computed Initial Rates of Vaporization at  $2800^{\circ}\text{K}$ .



Computed initial rate of vaporization at  $3000^\circ\text{K}$ .

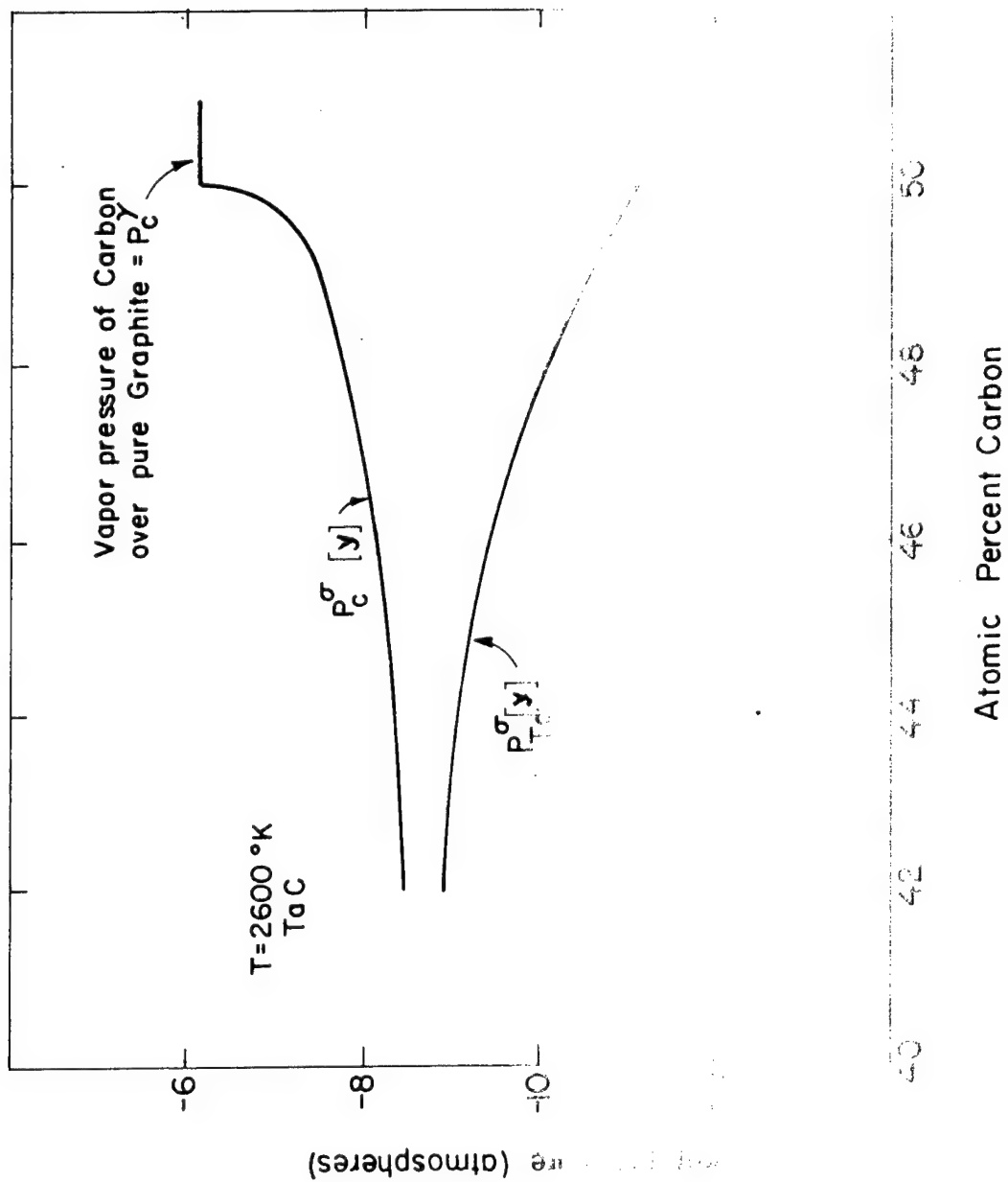
Fig. 20 - Computed Initial Rates of Vaporization at  $3000^\circ\text{K}$ .

In addition, Figs. 17 to 20 repeat the  $\dot{v}$  [x] curves corresponding to 49% carbon i.e.  $y = 0.49$ . The reason for including the 49% curves is as follows:

Fig. 18 corresponding to  $2600^{\circ}\text{K}$  shows the results of vaporization data obtained by Deadmore at NASA on a series of stoichiometric (50% carbon) Ta-Hf-C alloys at  $2600^{\circ}\text{K}$ . These data are the results of 2-4 hour exposures representing "average vaporization" rates. The Ta-C results at  $2600^{\circ}\text{K}$  yield  $\dot{v} = 10^{-7}$  gms/cm<sup>2</sup> sec. which corresponds to a carbon level of 49%. Reference to Fig. 21 (which is the companion to Figs. 14 and 15 of ref. 2) shows that if the experimental exposure time were extended, the carbon level should decrease resulting in a further reduction in  $\dot{v}$ . This reduction of  $\dot{v}$  is shown in Figs. 6-8. However, the exposure periods in the TaC case was only long enough to produce an "average  $\dot{v}$ " which corresponds to an "instantaneous  $\dot{v}$ " characteristic of a 49%C alloy. Consequently as a first approximation to the task of comparing the computed values of  $\dot{v}$  with those observed, 49%C is taken as a lower limit controlled by the exposure time. Thus we expect that the "quasi-congruency"  $\dot{v}$  corresponding to values of  $y_c$  equal to or greater than 0.49 can be reached and maintained, but that the exposure time required for reaching lower values of  $y_c$  is longer than the experimental exposure time.

On this basis the "computed"  $\dot{v}$ , x, y curve consists of two branches:

- a)  $\dot{v} = \dot{v} [y_c, x_c, T]$  for  $y_c \geq 0.49$
- b)  $\dot{v} = \dot{v} [x, y = 0.49, T]$  for  $y_c < 0.49$



**Fig. 21 -** Calculated compositional dependence of the vapor pressures of Ta and C over the  $TaC^\sigma$  phase at  $2600 \text{ } ^\circ\text{K}$ .



#### IV CONCLUSIONS

The Schottky-Wagner model of a two-sublattice compound has been applied to the (Ta-Hf)C ternary compound and detailed in a form which is sufficiently explicit to permit computation of the individual vapor pressures of the elemental components and the rates of Langmuir evaporation for a wide range of compositions and temperatures. As was pointed out at the end of Section III, the computed rates of vaporization represent initial or instantaneous rates corresponding to a fixed composition. The exception is pure HfC which exhibits congruent vaporization.

In view of the level of agreement between the predictions and available data (see Fig. 18) the thermodynamic description of this ternary compound detailed in Section III appears quite adequate. The comparison of predicted initial rates of vaporization and observed rates (Fig. 18) shows fair agreement, the maximum difference, being a factor of 5 at 2600°K. In addition, the appearance of a minimum in the rate of evaporation which is observed near 4TaC: HfC is also predicted by the present calculations. Under these circumstances the general agreement between the computed results and the experimental behavior appears quite satisfactory.

The present computations could be further improved by converting the equations for calculating initial rates of vaporization, Eqs. (42)-(44), to integrated rates of vaporization which might be compared with experimental data on a more realistic basis. The latter problem is an exercise in mathematics which does not require additional thermodynamic study.

For example consider a case similar to the binary Ta-C at 2600°K (Fig. 25) where the pressure of metal is lower than the pressure of carbon. If a composition corresponding to  $y = 0.5$  ( $x = 0$ ) is prepared, heated to 2600°K and allowed to vaporize, then the initial rates of vaporization  $\dot{v}_C$  and  $\dot{v}_{Ta}$  can be computed from Eqs. (35-44) as in Fig. (5). However, as the time of exposure increases from  $t = 0$  to some finite time  $t$ , the composition will change. Since congruency conditions are absent, the specimen will loose carbon and  $y$  will decrease from the initial value,  $y_0 = 0.5$  at  $t = 0$ , to a value  $y[t]$  which is less than 0.5. As this compositional change occurs,  $\dot{v}_C[t = 0]$  will decrease (see Figs. 6, 7, 8).

In order to compute  $\dot{v}_C[t]$  and  $\dot{v}[t]$  for this case, the changes in composition must be taken into account. To illustrate the indicated procedure, consider a case similar to the TaC situation at 2600°K, i.e.  $x = 0$ , where  $y = 0.5$  at  $t = 0$ . Since the vapor pressure of tantalum is much lower than that of carbon initially, let us assume that  $\dot{v}_{Ta}[t] \approx 0$  as a first approximation. If  $w_C$  and  $w_{Ta}$  are the number of grams of carbon in the specimen then

$$y[t] = w_C[t] / (w_{Ta}^0 M_C M_{Ta}^{-1} + w_C[t]) \quad (47)$$

where  $w_{Ta}[t] = \text{constant} = w_{Ta}^0$  and  $M_C$  and  $M_{Ta}^{-1}$  are the atomic weights of carbon and tantalum respectively; when  $t = 0$ ,  $y[t] = 0.5$ . Eq. (44) defines

$$\dot{v}_C[y] = - \frac{dw_C[y]}{dt} = 44.4 M_C^{1/2} T^{-1/2} p_C[y] = y(1-2y)^{-1} K_C[T] \quad (48)$$

where

$$K_C[T] = 44.4 M_C^{1/2} T^{-1/2} p_C^0 \exp[-F_{C+}/RT] \quad (49)$$

is constant at a given temperature. Substituting (48) into (47) yields

$$\frac{dw_C}{dt} = -K_C [T] (w_{Ta}^0 M_C w_C^{-1} M_{Ta}^{-1} - 1) \quad (50)$$

Integrating yields

$$w_C - w_C^0 = w_C^0 \ln y / (1-y) + t K_C \quad (51)$$

hence

$$(1-2y)/(1-y) + \ln y / (1-y) = -t K_C / w_C^0 \quad (52)$$

Eqs. (51) and (52) define  $y[t]$  and the carbon weight loss (i.e. the total weight loss since  $dw_{Ta}/dT$  is assumed equal to zero) as a function of  $y$  and time. Combination of Eqs. (51) and (52) yields

$$-\frac{(w_C - w_C^0)}{t} = \dot{v}_C[t] = \frac{K_C (1-2y)/(1-y)}{(2y-1)/(1-y) + \ln (1-y)/y} \quad (54)$$

Eqs. 51 to 54 can be simplified by considering times short enough so that the  $\ln(1-y)/y$  can be expanded in series approximation. This procedure yields

$$t \approx \frac{w_C^0}{2K_C} \left( \frac{2y-1}{1-y} \right)^2 \quad (55)$$

and

$$\dot{v}_C[t] \approx 2K_C (1-y) (1-2y)^{-1} \quad (56)$$

or

$$\dot{v}_C[t] = (2w_C^0 K_C / t)^{1/2} \quad (57)$$

Thus on the basis of the present simplified expression, the rate of evaporation is found to vary inversely as the square root of the time. A more complete solution of the problem would require relaxation of the assumption that

$$dw_{Ta}/dt = 0.$$

## REFERENCES

1. Kaufman, L. and Clougherty, E.V., "Thermodynamic Factors Controlling the Stability of Solid Phases at High Temperatures and Pressures". Metallurgy at High Temperature and High Pressure, A.I.M.E. Symposium, Dallas 1963 to be published by Gordon and Breech, N.Y. N.Y. 1964 Edited by K. Gschneider, N. Parlee and M. Hepworth.
2. Kaufman, L., Bernstein, H. and Sarney, A., "Thermodynamics of Interstitial Solid Solutions and Refractory Compounds". A.S.D.-TR-61-445 pt III ManLabs Report November 1963 under contract No. AF33(657)-9826 with Air Force Materials Laboratory, Dayton, Ohio.

*"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."*

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

## NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

**TECHNICAL REPORTS:** Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

**TECHNICAL NOTES:** Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

**TECHNICAL MEMORANDUMS:** Information receiving limited distribution because of preliminary data, security classification, or other reasons.

**CONTRACTOR REPORTS:** Technical information generated in connection with a NASA contract or grant and released under NASA auspices.

**TECHNICAL TRANSLATIONS:** Information published in a foreign language considered to merit NASA distribution in English.

**TECHNICAL REPRINTS:** Information derived from NASA activities and initially published in the form of journal articles.

**SPECIAL PUBLICATIONS:** Information derived from or of value to NASA activities but not necessarily reporting the results of individual NASA-programmed scientific efforts. Publications include conference proceedings, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

*Details on the availability of these publications may be obtained from:*

SCIENTIFIC AND TECHNICAL INFORMATION DIVISION  
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546